DIFFERENTITATION

Single Correct Answer Type

1. If $e^x = \frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}$ and $\tan\frac{y}{2} = \sqrt{\frac{1-t}{1+t}}$, then $\frac{dy}{dx}$ at $t = \frac{1}{2}$ is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 0 d) None of these

2. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ is equal to $(0 < x < \pi/2)$

a) $\sec^2 x$ b) $-\sec^2\left(\frac{\pi}{4}-x\right)$ c) $\sec^2\left(\frac{\pi}{4}+x\right)$ d) $\sec^2\left(\frac{\pi}{4}-x\right)$

3. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative, $D^*F(x)$ by the formula
$D^*(x) = \lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$, where $f^2(x)$ means
$[f(x)]^2$. If $f(x) = x\log x$, then
$D^*f(x)|_{x=e}$ has the value

a) e b) $2e$ c) $4e$ d) None of these

4. The differential coefficient of $f(\log_e x)$ with respect to x, where $f(x) = \log_e x$, is

a) $\frac{x}{\log_e x}$ b) $\frac{1}{x}\log_e x$ c) $\frac{1}{x\log_e x}$ d) None of these

5. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right)$, then $(x^2+1)\frac{dy}{dx}+xy+1 =$

a) 0 b) 1 c) 2 d) None of these

6. $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}$, where $0 < x < \pi$, is

a) $-1/2$ b) 0 c) 1 d) -1

7. $\frac{d^2x}{dy^2}$ equals

a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

8. If $f(x) = \sqrt{1-\sin 2x}$, then $f'(x)$ is equal to

a) $-(\cos x+\sin x)$, for $x \in (\pi/4, \pi/2)$ b) $\cos x+\sin x$, for $x \in (0, \pi/4)$
c) $-(\cos x+\sin x)$,for $x \in (0, \pi/4)$ d) $\cos x-\sin x$, for $x \in (\pi/4, \pi/2)$

9. If $x = t^2, y = t^3$, then $\frac{d^2y}{dx^2} =$

a) $\frac{3}{2}$ b) $\frac{3}{(4t)}$ c) $\frac{3}{2(t)}$ d) $\frac{3t}{2}$

10. If $y = x^2 + \frac{1}{x^2+\frac{1}{x^2+\frac{1}{x^2+\cdots\infty}}}$, then $\frac{dy}{dx}$ is

a) $\frac{2xy}{2y-x^2}$ b) $\frac{xy}{y+x^2}$ c) $\frac{xy}{y-x^2}$ d) $\frac{2xy}{2+\frac{x^2}{y}}$

11. Let $f(x) = \lim_{h\to 0}\frac{(\sin(x+h))^{\text{In}(x+h)}-(\sin x)^{\text{In}\,x}}{h}$, then $f\left(\frac{\pi}{2}\right)$ is

a) Equal to 0 b) Equal to 1 c) $\text{In}\frac{\pi}{2}$ d) Non-existent

12. If $y^2 = P(x)$, a polynomial of degree 3, then
$2\frac{d}{dx}\left(y^2\frac{d^2y}{dx^2}\right) =$

a) $P'''(x) + P'(x)$ b) $P''(x) + P'''(x)$ c) $P(x) + P'''(x)$ d) A constant

13. If $y = \log_{\sin x}(\tan x)$, then $\left(\frac{dy}{dx}\right)_{\pi/4}$ is equal to

a) $\dfrac{4}{\log 2}$ b) $-4\log 2$ c) $\dfrac{-4}{\log 2}$ d) None of these

14. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y$ is equal to

a) $m^2(ae^{mx} - be^{-mx})$ b) 1 c) 0 d) None of these

15. If $y = \tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at $x = 0$ is

a) 1 b) 2 c) In 2 d) None of these

16. If $(\sin x)(\cos y) = 1/2$, then d^2y/dx^2 at $(\pi/4, \pi/4)$ is

a) -4 b) -2 c) -6 d) 0

17. If $u = x^2 + y^2$ and $x = s + 3t, y = 2s - t$, then $\frac{d^2u}{ds^2}$ equals to

a) 12 b) 32 c) 36 d) 10

18. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is

a) $\cos\dfrac{\pi}{4}$ b) $\sin\dfrac{\pi}{2}$ c) $\sin\dfrac{\pi}{6}$ d) $\cos\dfrac{\pi}{3}$

19. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1 - x^2)\frac{dy}{dx}$ is equal to

a) $x + y$ b) $1 + xy$ c) $1 - xy$ d) $xy - 2$

20. Let $h(x)$ be differentiable for all x and let $f(x) = (kx + e^x)h(x)$, where k is some constant. If $h(0) = 5, h'(0) = -2$ and $f'(0) = 18$, then the value of k is

a) 5 b) 4 c) 3 d) 2.2

21. If $x = \log p$ and $y = \frac{1}{p}$, then

a) $\dfrac{d^2y}{dx^2} - 2p = 0$ b) $\dfrac{d^2y}{dx^2} + y = 0$ c) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$ d) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = 0$

22. $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{\sqrt{x}(3-x)}{1-3x}\right)\right] =$

a) $\dfrac{1}{2(1+x)\sqrt{x}}$ b) $\dfrac{3}{(1+x)\sqrt{x}}$ c) $\dfrac{2}{(1+x)\sqrt{x}}$ d) $\dfrac{3}{2(1+x)\sqrt{x}}$

23. If $y = \sin x + e^x$, then $\frac{d^2y}{dy^2} =$

a) $(-\sin x + e^x)^{-1}$ b) $\dfrac{\sin x - e^x}{(\cos x + e^x)^2}$ c) $\dfrac{\sin x - e^x}{(\cos x + e^x)^3}$ d) $\dfrac{\sin x + e^x}{(\cos x + e^x)^3}$

24. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{x}{y}$ b) $\dfrac{y}{x^2}$ c) $\dfrac{x^2 - y^2}{x^2 + y^2}$ d) $\dfrac{y}{x}$

25. If $y = \frac{(a-x)\sqrt{a-x}-(b-x)\sqrt{x-b}}{\sqrt{a-x}+\sqrt{x-b}}$, then $\frac{dy}{dx}$ wherever it is defined is

a) $\dfrac{x + (a+b)}{\sqrt{(a-x)(x-b)}}$ b) $\dfrac{2x - a - b}{2\sqrt{a-x}\sqrt{x-b}}$ c) $-\dfrac{(a+b)}{2\sqrt{(a-x)(x-b)}}$ d) $\dfrac{2x + (a+b)}{2\sqrt{(a-x)(x-b)}}$

26. If $f(x) = |\sin x - |\cos x||$, then the value $f'(x)$ at $x = 7\pi/6$ is

a) Positive b) $\dfrac{1-\sqrt{3}}{2}$ c) 0 d) None of these

27. If $f(0) = 0, f'(0) = 2$, then the derivative of
$y = f(f(f(f(x))))$ at $x = 0$ is

a) 2 b) 8 c) 16 d) 4

28. If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx} =$
a) $(\sin x)^{\tan x}(1 + \sec^2 x \log \sin x)$
b) $\tan x\,(\sin x)^{\tan x - 1}. \cos x$
c) $(\sin x)^{\tan x} \sec^2 x \log \sin x$
d) $\tan x (\sin x)^{\tan x - 1}$

29. If $f(x) = \sin^{-1} \cos x$, then the value of $f(10) + f'(10)$ is
a) $11 - \frac{7\pi}{2}$
b) $\frac{7\pi}{2} - 11$
c) $\frac{5\pi}{2} - 11$
d) None of these

30. The nth derivative of xe^x vanishes when
a) $x = 0$
b) $x = -1$
c) $x = -n$
d) $x = n$

31. The first derivative of the function
$\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right) + x^x\right]$ with respect to x at $x = 1$ is
a) 3/4
b) 0
c) 1/2
d) −1/2

32. If $y = ax^{n+1} + bx^{-n}$, then $x^2 \frac{d^2y}{dx^2}$ is equal to
a) $n(n-1)y$
b) $n(n+1)y$
c) ny
d) $n^2 y$

33. A function f satisfies the condition, $f(x) = f'(x) + f''(x) + f'''(x) + \cdots$ where $f(x)$ is a differentiable function indefinitely and dash denotes the order of derivative. If $f(0) = 1$, then $f(x)$ is
a) $e^{x/2}$
b) e^x
c) e^{2x}
d) e^{4x}

34. If $f(a) = 2, f'(a) = 1, \mathrm{g}(a) = -1, \mathrm{g}'(a) = 2,$ then the value of $\lim_{x \to a} \frac{\mathrm{g}(x)f(a) - \mathrm{g}(a)f(x)}{x - a}$ is
a) −5
b) $\frac{1}{5}$
c) 5
d) None of these

35. If $f(x) = \sqrt{1 + \cos^2(x)^2}$, then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is
a) $\sqrt{\pi}/6$
b) $-\sqrt{(\pi/6)}$
c) $1/\sqrt{6}$
d) $\pi/\sqrt{6}$

36. If $y^{1/m} = (x + \sqrt{1 + x^2})$, then $(1 + x^2)y_2 + xy_1$ is (where y_r represents rth derivative of y w.r.t. x)
a) $m^2 y$
b) my^2
c) $m^2 y^2$
d) None of these

37. A function f, defined for all positive real numbers, satisfies the equation $f(x^2) = x^3$ for every $x > 0$. Then the value of $f'(4) =$
a) 12
b) 3
c) 3/2
d) Cannot be determined

38. If $y = x^{(x^x)}$, then $\frac{dy}{dx}$ is
a) $y[x^x(\log ex)\log x + x^x]$
b) $y[x^x(\log ex)\log x + x]$
c) $y[x^x(\log ex)\log x + x^{x-1}]$
d) $y[x^x(\log_e x)\log x + x^{x-1}]$

39. $\frac{d}{dx} \cos^{-1}\sqrt{\cos x}$ is equal to
a) $\frac{1}{2}\sqrt{1 + \sec x}$
b) $\sqrt{1 + \sec x}$
c) $-\frac{1}{2}\sqrt{1 + \sec x}$
d) $-\sqrt{1 + \sec x}$

40. If $y = a \sin x + b \cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a
a) function of x
b) function of y
c) function of x and y
d) constant

41. If $y = \sin px$ and y_n is the nth derivative of y, the
$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ is
a) 1
b) 0
c) −1
d) None of these

42. Let $\mathrm{g}(x)$ be the inverse of an invertible function $f(x)$ which is differentiable for all real x, then $\mathrm{g}''(f(x))$ equals

a) $-\dfrac{f''(x)}{(f'(x))^3}$
b) $\dfrac{f'(x)f''(x)-(f'(x))^3}{f'(x)}$
c) $\dfrac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$
d) None of these

43. $\dfrac{d^n}{dx^n}(\log x) =$

a) $\dfrac{(n-1)!}{x^n}$ b) $\dfrac{n!}{x^n}$ c) $\dfrac{(n-2)!}{x^n}$ d) $(-1)^{n-1}\dfrac{(n-1)!}{x^n}$

44. If $f(x) = |x^2 - 5x + 6|$, then $f'(x)$ equals

a) $2x - 5$ for $2 < x < 3$ b) $5 - 2x$ for $2 < x < 3$ c) $2x - 5$ for $x > 2$ d) $5 - 2x$ for $x < 3$

45. If $u = f(x^3), v = g(x^2), f'(x) = \cos x$ and $g'(x) = \sin x$, then $\frac{du}{dv}$ is

a) $\dfrac{3}{2}\, x \cos x^3 \operatorname{cosec} x^2$ b) $\dfrac{2}{3} \sin x^3 \sec x^2$ c) $\tan x$ d) None of these

46. If $y = x - x^2$, then the derivative of y^2 with respect to x^2 is

a) $1 - 2x$ b) $2 - 4x$ c) $3x - 2x^2$ d) $1 - 3x + 2x^2$

47. If $f(x) = 2\sin^{-1}\sqrt{1-x} + \sin^{-1}\left(2\sqrt{x(1-x)}\right)$,where $x \in \left(0, \frac{1}{2}\right)$, then $f'(x)$ is

a) $\dfrac{2}{\sqrt{x(1-x)}}$ b) zero c) $-\dfrac{2}{\sqrt{x(1-x)}}$ d) π

48. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right) = \frac{5f(x)-3f(y)}{2}$ $\forall x, y \epsilon R$, and $f(0) = 3$ and $f'(0) = 2$, then the period of $\sin(f(x))$ is

a) 2π b) π c) 3π d) 4π

49. The function $f(x) = e^x + x$, being differentiable and one to one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point $f(\log 2)$ is

a) $\dfrac{1}{\text{In } 2}$ b) $\dfrac{1}{3}$ c) $\dfrac{1}{4}$ d) None of these

50. $\frac{d}{dx}\left[\sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ is equal to

a) -1 b) $\dfrac{1}{2}$ c) $-\dfrac{1}{2}$ d) 1

51. If $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ $(0 < x < \pi/2)$, then $\frac{dy}{dx} =$

a) $\dfrac{1}{2}$ b) $\dfrac{2}{3}$ c) 3 d) 1

52. If $y = \cos^{-1}\left(\frac{5\cos x - 12\sin x}{13}\right)$, where$x \in \left(0, \frac{\pi}{2}\right)$, then$\frac{dy}{dx}$ is

a) 1 b) -1 c) 0 d) None of these

53. If $y = \tan^{-1}\sqrt{\frac{x+1}{x-1}}$, then $\frac{dy}{dx}$ is

a) $\dfrac{-1}{2|x|\sqrt{x^2-1}}$ b) $\dfrac{-1}{2x\sqrt{x^2-1}}$ c) $\dfrac{1}{2x\sqrt{x^2-1}}$ d) None of these

54. The nth derivative of the function $f(x) = \frac{1}{1-x^2}$ (where $x \in (-1,1)$) at the point $x = 0$ where n is even is

a) 0 b) $n!$ c) n^nC_2 d) 2^nC_2

55. If $y = \sqrt{\frac{1-x}{1+x}}$, then $(1-x^2)\frac{dy}{dx}$ is equal to

a) y^2 b) $1/y$ c) $-y$ d) $-y/x$

56. If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, than $\frac{d^2y}{dx^2}$ is

a) 2 b) 1 c) 0 d) -1

57. If $f(x) = x + \tan x$ and f is inverse of g, then g'(x) equals

a) $\frac{1}{1 + [g(x) - x]^2}$ b) $\frac{1}{2 - [g(x) - x]2}$ c) $\frac{1}{2 + [g(x) - x]^2}$ d) None of these

58. If g is the inverse function of f and $f'(x) = \sin x$, then $g'(x)$ is

a) cosec $\{g(x)\}$ b) $\sin\{g(x)\}$ c) $-\frac{1}{\sin\{g(x)\}}$ d) None of these

59. If $f(x) = |\log_e|x||$, then $f'(x)$ equal

a) $\frac{1}{|x|}$, where $x \neq 0$ b) $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

c) $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$ d) $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

60. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is

a) e^x b) $-\frac{e^x}{(1 + e^x)^3}$ c) $-\frac{e^x}{(1 + e^x)^2}$ d) $\frac{-1}{(1 + e^x)^3}$

61. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$, then $\frac{dy}{dx}$ is equal to

a) y b) $y + \frac{x^n}{n!}$ c) $y - \frac{x^n}{n!}$ d) $y - 1 - \frac{x^n}{n!}$

62. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$ is

a) 1/8 b) 1/4 c) 1/2 d) 1

63. If $y = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is

a) $\frac{1 - \sqrt{3}}{2}$ b) 0 c) $\frac{1}{2}(\sqrt{3} - 1)$ d) None of these

64. Let $f(x)$ be a quadratic expression which is positive for all the real values of x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x,

a) $g(x) < 0$ b) $g(x) > 0$ c) $g(x) = 0$ d) $g(x) \geq 0$

65. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the product ab is

a) 25 b) 9 c) -15 d) -9

66. If $x = \emptyset(t), y = \Psi(t)$, then $\frac{d^2y}{dx^2}$ is

a) $\frac{\phi'\Psi'' - \Psi'\phi''}{(\phi')^2}$ b) $\frac{\phi'\Psi'' - \Psi'\phi''}{(\phi')^3}$ c) $\frac{\phi''}{\Psi''}$ d) $\frac{\Psi''}{\phi''}$

67. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n$, then $\frac{dy}{dx}$ is

a) $\frac{ny}{\sqrt{x^2 + a^2}}$ b) $-\frac{ny}{\sqrt{x^2 + a^2}}$ c) $\frac{nx}{\sqrt{x^2 + a^2}}$ d) $-\frac{nx}{\sqrt{x^2 + a^2}}$

68. Let $g(x) = \log f(x)$, where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = xf(x)$. Then, for $N = 1,2,3, \ldots, g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$ is equal to

a) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N - 1)^2}\right\}$ b) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N - 1)^2}\right\}$

c) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N + 1)^2}\right\}$ d) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N + 1)^2}\right\}$

69. Let $y =$ In $(1 + \cos x)^2$, then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals

a) 0 b) $\frac{2}{1 + \cos x}$ c) $\frac{4}{(1 + \cos x)}$ d) $\frac{-4}{(1 + \cos x)^2}$

70. Let $u(x)$ and $v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7$. If $\frac{u\prime(x)}{v\prime(x)} = p$ and $\left(\frac{u(x)}{v(x)}\right) = q$, then $\frac{p+q}{p-q}$ has the value equal to

a) 1 b) 0 c) 7 d) -7

71. If $f''(x) = -f(x)$, where $f(x)$ is a continuous double differentiable function and $g(x) = f'(x)$. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and $F(5) = 5$ then $F(10)$ is

a) 0 b) 5 c) 10 d) 25

72. $x = t\cos t, y = t + \sin t$, then $\frac{d^2x}{dy^2}$ at $t = \frac{\pi}{2}$ is

a) $\dfrac{\pi + 4}{2}$ b) $-\dfrac{\pi + 4}{2}$ c) -2 d) None of these

73. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at $x = 1$ has the value equal to

a) 19 b) 9 c) 17 d) 14

74. If $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{ay}{x\sqrt{a^2 - x^2}}$ b) $\dfrac{ay}{\sqrt{a^2 - x^2}}$ c) $\dfrac{ay}{x\sqrt{x^2 - a^2}}$ d) None of these

75. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x)$ is equal to

a) $2^{20}(\cos 2x - 2^{20}\cos 3x)$ b) $2^{20}(\cos 2x + 2^{20}\cos 4x)$

c) $2^{20}(\sin 2x + 2^{20}\sin 4x)$ d) $2^{20}(\sin 2x - 2^{20}\sin 4x)$

76. Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is

a) $\dfrac{5}{2}t$ b) $20t^8$ c) $\dfrac{5}{16t^6}$ d) None of these

77. If $f(x) = x^4\tan(x^3) - x\,\text{In}\,(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is

a) 0 b) 6 c) 12 d) 24

78. $f(x) = e^x - e^{-x} - 2\sin x - \frac{2}{3}x^3$, then the least value of n for which $\left.\frac{d^n}{dx^n} f(x)\right|_{x=0}$ is non-zero is

a) 5 b) 6 c) 7 d) 8

79. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$, then $g'(f(c))$ equals

a) $f'(c)$ b) $\dfrac{1}{f'(c)}$ c) $f(c)$ d) None of these

80. The derivative of $y = (1-x)(2-x)\dots(n-x)$ at $x = 1$ is

a) 0 b) $(-1)(n-1)!$ c) $n! - 1$ d) $(-1)^{n-1}(n-1)!$

81. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{d^2y}{dx^2}$ is

a) $\dfrac{h^2 - ab}{(hx + by)^2}$ b) $\dfrac{ab - h^2}{(hx + by)^2}$ c) $\dfrac{h^2 + ab}{(hx + by)^2}$ d) None of these

82. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots\infty}}}$, then $\frac{dy}{dx}$ is

a) $\dfrac{x}{2y - 1}$ b) $\dfrac{x}{2y + 1}$ c) $\dfrac{1}{x(2y - 1)}$ d) $\dfrac{1}{x(1 - 2y)}$

83. If $y = x\log\left(\frac{x}{a+bx}\right)$, then $x^3\frac{d^2y}{dx^2}$ equals to

a) $x\dfrac{dy}{dx} - y$ b) $\left(x\dfrac{dy}{dx} - y\right)^2$ c) $y\dfrac{dy}{dx} - x$ d) $\left(y\dfrac{dy}{dx} - x\right)^2$

84. If graph of $y = f(x)$ is symmetrical about y-axis and that of $y = g(x)$ is symmetrical about the origin. If $h(x) = f(x)\cdot g(x)$, then $\frac{d^3h(x)}{dx^3}$ at $x = 0$ is

a) Cannot be determined b) $f(0)\cdot g(0)$

c) 0 d) None of these

85. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then $\frac{dy}{dx}$ is equal to

a) $\sqrt{\frac{1-x^2}{1-y^2}}$ b) $\sqrt{\frac{1-y^2}{1-x^2}}$ c) $\sqrt{\frac{x^2-1}{1-y^2}}$ d) $\frac{y^2-1}{1-x^2}$

86. If $y^2 = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is

a) a constant b) a function of x only c) a function of y only d) a function of x and y

87. If $f'(x) = \sqrt{2x^2-1}$ and $y = f(x^2)$, then $\frac{dy}{dx}$ at $x = 1$ is

a) 2 b) 1 c) -2 d) None of these

88. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3 y \frac{dy}{dx} =$

a) 0 b) 1 c) -1 d) None of these

89. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is

a) 5 b) 10 c) 0 d) 15

90. If $y = \cos^{-1}(\cos x)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{4}$ is

a) 1 b) -1 c) $\frac{1}{\sqrt{2}}$ d) None of these

91. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

a) p
b) $p - p^3$
c) $p + p^3$
d) Independent of p

Multiple Correct Answers Type

92. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is

a) $f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$ b) $f_n(x)f_{n-1}(x)$
c) $f_n(x)f_{n-1}(x) \dots f_2(x).f_1(x)$ d) None of these

93. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is

a) $\frac{-2}{1+x^2}$ for all x b) $\frac{-2}{1+x^2}$ for all $|x| < 1$ c) $\frac{2}{1+x^2}$for $|x| > 1$ d) None of these

94. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ b) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$ c) $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$ d) $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

95. $f(x) = |x^2 - 3|x| + 2|$, then which of the following is/are true

a) $f'(x) = 2x - 3$ for $x \in (0,1) \cup (2, \infty)$
b) $f'(x) = 2x + 3$ for $x \in (-\infty, -2) \cup (-1,0)$
c) $f'(x) = -2x - 3$ for $x \in (-2, -1)$
d) None of these

96. If $f(x) = \sin^{-1}(\sin x)$, then

a) $f'\left(\frac{3\pi}{4}\right) = 1$ b) $f'\left(\frac{5\pi}{4}\right) = -1$
c) $f'\left(\frac{\pi}{2}\right)$ does not exist d) $f'(\pi)$ does not exist

97. If $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$ and $\frac{dy}{dx} = ax + b$, then the value of $a - b$ is

a) $\cot\frac{\pi}{8}$ b) $\cot\frac{\pi}{12}$ c) $\tan\frac{5\pi}{12}$ d) $\tan\frac{5\pi}{8}$

98. If 1 is a twice repeated root of the equation $ax^3 + bx^2 + bx + d = 0$, then
a) $a = b = d$ b) $a + b = 0$ c) $b + d = 0$ d) $a = d$

99. Differential coefficient of $\sin^{-1} x$ w. r. t. $\sin^{-1}(3x - 4x^3)$ is
a) $\frac{1}{3}$ if $-\frac{\pi}{8} < x < \frac{\pi}{8}$ b) 3 if $\frac{-\pi}{8} < x < \frac{\pi}{8}$ c) $\frac{1}{3}$ if $-\frac{\pi}{9} < x < \frac{\pi}{9}$ d) 3 if $\frac{-\pi}{9} < x < \frac{\pi}{9}$

100. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ and $y(1) = 1$, then
a) $y'(1) = 4/3$ b) $y''(1) = -4/3$ c) $y''(1) = -8\frac{22}{27}$ d) $y'(1) = 2/3$

101. If 1 is a twice repeated root of the equation $ax^3 + bx^2 + bx + d = 0$, then
a) $a = b = d$ b) $a + b = 0$ c) $b + d = 0$ d) $a = d$

102. Let $f(x) = \frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}x$, then
a) $f'(10) = 1$
b) $f'(3/2) = -1$
c) Domain of $f(x)$ is $x \geq 1$
d) Range of $f(x)$ is $(-2, -1] \cup (2, \infty)$

103. If $F(x) = f(x)g(x)$ and $f'(x)g'(x) = c$, then
a) $F' = c\left[\frac{f}{f'} + \frac{g}{g'}\right]$
b) $\frac{F''}{F} = \frac{f''}{f} + \frac{g''}{g} + \frac{2c}{fg}$
c) $\frac{F'''}{F} = \frac{f'''}{f} + \frac{g'''}{g}$
d) $\frac{F'''}{F''} = \frac{f'''}{f''} + \frac{g''}{g''}$

104. Which of the following is/are true?
a) $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$, where $x \in (0, \pi)$, is -1
b) $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$, where $x \in (\pi, 2\pi)$, is 1
c) $\frac{dy}{dx}$ for $y = \cos^{-1}(\sin x)$, where $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, is -1
d) $\frac{dy}{dx}$ for $y = \cos^{-1}(\sin x)$, where $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, is -1

105. If $f(x - y), f(x)f(y)$ and $f(x + y)$ are in A.P. for all x, y and $f(0) \neq 0$, then
a) $f(4) = f(-4)$ b) $f(2) + f(-2) = 0$ c) $f'(4) + f'(-4) = 0$ d) $f'(2) = f'(-2)$

106. If $y = x^{(\log x)^{\log(\log, x)}}$, then $\frac{dy}{dx}$ is
a) $\frac{y}{x}\left((\text{In}x^{\infty x-1}) + 2\text{In}\, x\ \text{In}\,(\text{In}x)\right)$
b) $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$
c) $\frac{y}{x\ \text{In}\, x}[(\text{In}x)^2 + 2\text{In}(\text{In}\, x)]$
d) $\frac{y}{x}\frac{\log y}{\log x}[2\log(\log x) + 1]$

107. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}$, then $\frac{dy}{dx}$ is equals to
a) $\frac{1}{2y - 1}$ b) $\frac{x}{x + 2y}$ c) $\frac{1}{\sqrt{1 + 4x}}$ d) $\frac{y}{2x + y}$

108. Let $f(t) = \text{In}\, t$.Then, $\frac{d}{dx}\left\{\int_{x^2}^{x^3} f(t)dt\right\}$
a) has a value 0, when $x = 0$
b) Has a value 0, when $x = 1, x = \frac{4}{9}$
c) Has a value $9e^2 - 4e$,when $x = e$
d) Has a differential coefficient $27e - 8$, when $x = e$

109. $f: R^+ \to R$ be a continuous function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y) \forall x, y \in R^+$. If $f'(1) = 1$, then
a) $'f'$ is unbounded b) $\lim_{x\to 0} f\left(\frac{1}{x}\right) = 0$ c) $\lim_{x\to 0}\frac{f(1+x)}{x} = 1$ d) $\lim_{x\to 0} x.f(x) = 0$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 110 to 109. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

110

Statement 1: If $u = f(\tan x), v = g(\sec x)$ and $f'(1) = 2$, $g'(\sqrt{2})$=4,then $\left(\frac{du}{dv}\right)_{x=\pi/4} = \frac{1}{\sqrt{2}}$

Statement 2: If $u = f(x), v = g(x)$, then the derivation of f with respect to g is $\frac{du}{dv} = \frac{du/dx}{dv/dx}$

111

Statement 1: Let $f: R \to R$ is a real-valued function $\forall\ x, y \in R$ such that $|f(x) - f(y)| \leq |x - y|^3$, then $f(x)$ is a constant function

Statement 2: If derivative of the function w.r.t. x is zero, then function is constant

112

Statement 1: If differentiable function $f(x)$ satisfies the relation $f(x) + f(x - 2) = 0\ \forall x \in R$, and if $\left(\frac{d}{dx}f(x)\right)_{x=a} = b$, then $\left(\frac{d}{dx}f(x)\right)_{a+4000} = b$

Statement 2: $f(x)$ is a periodic function with period 4

113

Statement 1: Derivative of $\sin^{-14}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0 < x < 1$

Statement 2: $\sin^{-1}\left(\frac{2x}{1+x^2}\right) \neq \cos^{-1}\left(\frac{1-x^2}{1-x^2}\right)$ for $-1 \leq x \leq 1$

114

Statement 1: For $f(x) = \sin x, f'(\pi) = f'(3\pi)$

Statement 2: For $f(x) = \sin x, f(\pi) = f(3\pi)$

115

Statement 1: For $x < 0, \frac{d}{dx}(\text{In}\ |x|) = -\frac{1}{x}$

Statement 2: For $x < 0, |x| = -x$

116 If for some differentiable function $f(\alpha) = 0$ and $f'(\alpha) = 0$

Statement 1: Then sign of $f(x)$ does not change in the neighborhood of $x = \alpha$

Statement 2: α is repeated root of $f(x) = 0$

117 Observe the following statements
Which of the following is correct?

Statement 1: I $f(x) = ax^{41} + bx^{-40} \Rightarrow \frac{f''(x)}{f(x)} = 1640x^{-2}$

Statement 2: II $\frac{d}{dx}\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{1}{1+x^2}$

118

Statement 1: If $e^{xy} + \log(xy) + \cos(xy) + 5 = 0$, then $\frac{dy}{dx} = -\frac{y}{x}$

Statement 2: $\frac{d}{dx}(xy) = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

119

Statement 1: Let $f(x) = x[x]$ and $[.]$ denotes greatest integral function, when x is not an integral, then rule for $f'(x)$ is given by $[x]$

Statement 2: $f'(x)$ does not exist for any $x \epsilon$ integer

120

Statement 1: If $f(x)$ is an odd function, then $f'(x)$ is an even function

Statement 2: If $f'(x)$ is an even function, then $f(x)$ is an odd function

121 Consider function $f(x)$ satisfies the relation, $f(x + y^3) = f(x) + f(y^3), \forall\, x, y \in R$ and differentiable for all x

Statement 1: If $f'(2) = a$, then $f'(-2) = a$

Statement 2: $f(x)$ is an odd function

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

122.

	Column-I		**Column- II**
(A)	$y = f(x)$ be givin by $x = t^5 - 5t^3 - 20t + 7$ and $y = 4t^3 - 3t^2 - 18t + 3$, then $-5 \times \frac{dy}{dx}$ at $t = 1$	(p)	0
(B)	$P(x)$ be a polynomial of degree 4, with $P(2) = -1, P'(2) = 0, P''(2)=2, P'''(2) = -12$ and $P^{iv}(2) = 24$, then $P''(3)$	(q)	-2
(C)	$y = \frac{1}{x}$, then $\frac{\frac{dy}{\sqrt{1+y^4}}}{\frac{dx}{1+x^4}}$	(r)	2
(D)	$f\left(\frac{2x+3y}{5}\right) = \frac{2f(x)+3f(y)}{5}$ and $f'(0) = p$ and $f(0) = q$, then $f''(0)$	(s)	-1

CODES :

A **B** **C** **D**

a)	P	q	r	s
b)	q	r	s	p
c)	t	r	s	p
d)	s	t	p	q

123.

	Column-I		**Column- II**
(A)	Differentiable function $f(x)$satisfies the relation $f(1-x)=f(1+x)$for all $x \in R$	(p)	Graph of $f'(x)$ is symmetrical about point (1,0)
(B)	Differentiable function $f(x)$ satisfies the relation $f(2-x)+f(x)=0$ for all $x \in R$	(q)	Graph of $f'(x)$ is symmetrical about line $x=1$
(C)	Differentiable function $f(x)$ satisfies the relation $f(x+2)+f(x)=0$ for all $x \in R$	(r)	$f'(-1)=f'(3)$
(D)	Differentiable function $f(x)$ satisfies the relation $f(x)+f(y)+f(x).f(y)=1$ for all x, y and $f(x)>0$	(s)	$f'(x)$ has period 4

CODES :

	A	**B**	**C**	**D**
a)	P	q,r	s,r	q,r
b)	q	p,s	t,r	q
c)	s	q,r	p,s	t
d)	s,r	t	q	p,q

124.

	Column-I		**Column- II**
(A)	$y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}=-\frac{2}{1+x^2}$	(p)	For $x<0$
(B)	$y=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, then $\frac{dy}{dx}=-\frac{1}{1+x^2}$	(q)	For $x>1$
(C)	$y=\left|e^{|x|}-e\right|$, then $\frac{dy}{dx}>0$	(r)	For $x<-1$
(D)	$u=\log|2x|, v=|\tan^{-1}x|$, then $du/dv>2$	(s)	For $-1<x<0$

CODES :

	A	**B**	**C**	**D**
a)	T	r	s,r	p
b)	s,p	t	p	q
c)	q,r	p,r,s	q,s	q,r
d)	q	r,q	s	p

125. Match the value of x in column II where derivative of the function in column I is negative

	Column-I		**Column- II**
(A)	$y = \|x^2 - 2\|x\|\|$	(p)	$(1, 2)$
(B)	$y = \|\log_e\|x\|\|$	(q)	$(-3, -2)$
(C)	$y = x[x/2]$, where [.] represents greatest integer function	(r)	$(-1,0)$
(D)	$y = \|\sin x\|$	(s)	$(0,1)$

CODES :

	A	**B**	**C**	**D**
a)	P,q,r	q,s	q,r	r
b)	s,r	t,s	p	q
c)	t	r	s	p
d)	t,q	p	r	s

Linked Comprehension Type

This section contain(s) 14 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 126 to -126

If $D^*f(x) = \lim_{h\to 0} \frac{f^2(x+h)-f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$

On the basis of above information, answer the following questions :

126. If $u = f(x), v = g(x)$, then the value of $D^*(u.v)$is

a) $(D^*u)v + (D^*v)u$ b) $u^2D^*v + v^2D^*u$ c) $D^*u + D^*v$ d) None of these

Paragraph for Question Nos. 127 to - 127

If $y = f(x)$be a differentiable function of x such whose second, third,..., nth derivatives exist.*ie*,nth derivative of y is denoted by

$y_n, \frac{d^ny}{dx^n}, D_y^n, y^n, f^n(x)$

$\Rightarrow \frac{d^ny}{dx^n} = \lim_{h\to 0} \frac{f^{n-1}(x+h) - f^{n-1}(x)}{h}$

On the basis of above information, answer the following question :

127. If $y = e^{3x+7}$,then the value of $y_n(0)$ is

a) 1 b) 3^n c) $3^n.e^7$ d) $3^n.e^7.7!$

Paragraph for Question Nos. 128 to - 128

$f(x)$ is a polynomial function $f: R \rightarrow R$ such that $f(2x) = f'(x)f''(x)$

128. The value of $f(3)$ is
a) 4 b) 12 c) 15 d) None of these

Paragraph for Question Nos. 129 to - 129

$f: R \rightarrow R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$

129. The value of $f(1)$ is
a) 2 b) 3 c) -1 d) 4

Paragraph for Question Nos. 130 to - 130

Repeated roots: if equation $f(x) = 0$, where $f(x)$ is a polynomial function, and if it has roots $\alpha, \alpha, \beta, \dots$ or α root is repeated root, then $f(x) = 0$ is equivalent to $(x-\alpha)^2(x-\beta)\dots = 0$, from which we can conclude that $f'(x) = 0$ or $2(x-\alpha)[(x-\beta)\dots] + (x-\alpha)^2(x-\beta)\dots]' = 0$ or $(x-\alpha)[2\{(x-\beta)\dots\} + (x-\alpha)\{(x-\beta)\dots\}'] = 0$ has root α. Thus, if α root occurs twice in equation , then it is common in equations $f(x) = 0$ and $f'(x) = 0$
Similarly, if α root occurs thrice in equation, then it is common in the equations $f(x) = 0, f'(x) = 0$ and $f''(x) = 0$

130. If $x - c$ is a factor of order m of the polynomial $f(x)$ of degree $n(1 < m < n)$, then $x = c$ is a root of the polynomial (where $f^r(x)$ represent rth derivative of $f(x)$ w.r.t.x)
a) $f^m(x)$ b) $f^{m-1}(x)$ c) $f''(x)$ d) None of these

Paragraph for Question Nos. 131 to - 131

Equation $x^n - 1 = 0, n > 1, n \in N$, has roots $1, a_1, a_2 \dots, a_{n-1}$

131. The value of $(1-a_1)(1-a_2)\dots(1-a_{n-1})$ is
a) $n^2/2$ b) n c) $(-1)^n n$ d) None of these

Paragraph for Question Nos. 132 to - 132

$f(x) = x^2 + x g'(1) + g''(2)$ and $g(x) = f(1)x^2 + x f'(x) + f''(x)$

132. The value of $f(3)$ is
a) 1 b) 0 c) -1 d) -2

Paragraph for Question Nos. 133 to - 133

$g(x+y) = g(x) + g(y) + 3xy(x+y) \forall\, x, y \in R$ and $g'(0) = -4$

133. Number of real roots of the equation $g(x) = 0$ is
   a) 2   b) 0   c) 1   d) 3

Paragraph for Question Nos. 134 to - 134

A curve is represented parametrically by the equations $x = f(t) = a^{\ln(b^t)}$ and $y = g(t) = b^{-\ln(a^t)} a, b > 0$ and $a \neq 1, b \neq 1$ where $t \in R$

134. Which of the following is not a correct expression for $\frac{dy}{dx}$?
   a) $\frac{-1}{f(t)^2}$ b) $-(g(t))^2$ c) $\frac{-g(t)}{f(t)}$ d) $\frac{-f(t)}{g(t)}$

Integer Answer Type

135. $\lim_{h \to 0} \frac{(e+h)^{\ln(e+h)} - e}{h}$ is
136. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $|ab|/3$ is
137. Let $y = \frac{2^{\log_2 1/4^x} - 3^{\log_{27}(x^2+1)^3} - 2x}{7^{4\log_{49} x} - x - 1}$ and $\frac{dy}{dx} = ax + b$, then the value of $a + b$ is
138. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ and $y' = 0$ at $x = 5$, then the value of a^2/b^2 is
139. Let $g(x) = \begin{cases} \frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x} & x \neq 0 \\ 0; & x = 0 \end{cases}$. If $g'(0)$ exists and is equal to non-zero value b, then $52\frac{b}{a}$ is equal to
140. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $|f'(-3)|$ equals
141. If $f'(x) = \phi(x)$ and $\phi'(x) = f(x)$ for all x. Also $f(3) = 5$ and $f'(3) = 4$. Then the value of $[f(10)]^2 - [\phi(10)]^2$ is
142. $y = f(x)$, where f satisfies the relation $f(x + y) = 2f(x) + xf(y) + y\sqrt{f(x)}\ \forall\, x, y \in R$ and $f'(0) = 0$, then $f(6)$ is equal to
143. Suppose that $f(0) = 0$ and $f'(0) = 2$, and let $g(x) = f\left(-x + f(f(x))\right)$. The value of $g'(0)$ is equal to
144. Let $z = (\cos x)^5$ and $y = \sin x$. Then the value of $2\frac{d^2z}{dy^2}$ at $x = \frac{2\pi}{9}$ is
145. A non-zero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$. If a is the leading coefficient of $f(x)$, then the value of $1/(2a)$ is
146. If function f satisfies the relation $f(x) \times f'(-x) = f(-x) \times f'(x)$ for all x, and $f(0) = 3$, now if $f(3) = 3$, then the value of $f(-3)$ is
147. A function is represented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}$, then the value of $\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is
148. Let $g(x) = f(x)\sin x$, where $f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $|g''(-\pi)|$ equals
149. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3$, then the value of $f'(3)$ is
150. Let $f(x) = (x-1)(x-2)(x-3)\ldots(x-n), n \in N$ and $f'(n) = 5040$, then the value of n is
151. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ and $g(x) = f^{-1}(x)$, then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is

152. If $x^3 + 3x^2 - 9x + c$ is one of the form $(x - \alpha)^2(x - \beta)$, then positive value of c is

: ANSWER KEY :

1)	a	2)	b	3)	c	4)	c	89)	a	90)	b	91)	d	1)	
5)	a	6)	a	7)	d	8)	c		a,c	2)	b,c	3)	a,c	4)	
9)	b	10)	a	11)	a	12)	c		a,b,c						
13)	c	14)	c	15)	d	16)	a	5)	b,c	6)	b,c	7)	b,c,d	8)	a
17)	d	18)	a	19)	b	20)	c	9)	a,c	10)	b,c,d	11)	a,b,d	12)	
21)	c	22)	d	23)	c	24)	d		a,b,c						
25)	b	26)	a	27)	c	28)	a	13)	a,b,c	14)	a,c	15)	b,d	16)	
29)	a	30)	c	31)	a	32)	b		a,c,d						
33)	a	34)	c	35)	b	36)	a	17)	b,c,d	18)	a,c,d	1)	a	2)	a
37)	b	38)	c	39)	a	40)	d		3)	a	4)	c			
41)	b	42)	a	43)	d	44)	b	5)	b	6)	d	7)	d	8)	a
45)	a	46)	d	47)	b	48)	b	9)	a	10)	a	11)	c	12)	a
49)	b	50)	b	51)	a	52)	a	1)	b	2)	a	3)	c	4)	a
53)	a	54)	b	55)	c	56)	c	1)	b	2)	c	3)	b	4)	d
57)	c	58)	a	59)	b	60)	b	5)	b	6)	b	7)	b	8)	d
61)	c	62)	b	63)	c	64)	b	9)	d	1)	2	2)	5	3)	3
65)	c	66)	b	67)	a	68)	a		4)	5					
69)	a	70)	a	71)	b	72)	b	5)	7	6)	2	7)	9	8)	9
73)	a	74)	a	75)	b	76)	c	9)	6	10)	5	11)	9	12)	3
77)	a	78)	c	79)	b	80)	b	13)	1	14)	2	15)	3	16)	8
81)	a	82)	c	83)	b	84)	a	17)	5	18)	5				
85)	b	86)	a	87)	a	88)	b								

: HINTS AND SOLUTIONS :

1 **(a)**

Let $t = \cos 2\theta$

Then $e^x = \frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\tan\theta}{1+\tan\theta} = \tan\left(\frac{\pi}{4}-\theta\right)$$

$$\tan\frac{y}{2} = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \tan\theta$$

At $t = \frac{1}{2}, \cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Then $x = \log\tan\frac{\pi}{12}, y = \frac{\pi}{3}$

Differentiating w.r.t. $\theta, e^x \frac{dx}{d\theta} = -\sec^2\left(\frac{\pi}{4}-\theta\right)$ and $\frac{1}{2}\sec^2\frac{y}{2}\frac{dy}{d\theta} = \sec^2\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\sec^2\theta\cos^2\frac{y}{2}}{-e^{-x}\sec^2\left(\frac{\pi}{4}-\theta\right)}$$

At $t = \frac{1}{2}$, i.e., $\theta = \frac{\pi}{6}, \frac{dy}{dx} = \frac{2\sec^2\frac{\pi}{6}\cos^2\frac{\pi}{6}}{-e^{-\log\tan\pi/12}\sec^2\frac{\pi}{12}}$

$$\therefore \frac{dy}{dx} = \frac{2}{-\cot\frac{\pi}{12}\sec^2\frac{\pi}{12}}$$

$$= -2\tan\frac{\pi}{12}\cos^2\frac{\pi}{12} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

2 **(b)**

$$y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1-\tan x}{1+\tan x}$$
$$= \tan\left(\frac{\pi}{4}-x\right)$$

$$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4}-x\right)$$

3 **(c)**

$$D^*(x) = \lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}(f(x+h)+f(x))$$

$$= 2f(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= 2f(x)\times f'(x)$$

$$\Rightarrow D^*(x\log x) = 2x\log x(1+\log x)$$

$$\Rightarrow D^*f(x)|_{x=e} = 4e$$

4 **(c)**

$f(\log_e x) = \log_e(\log_e x)$

$$\therefore \frac{df(\log_e x)}{dx} = \frac{1}{\log_e x}\times\frac{1}{x}$$

5 **(a)**

$$y\sqrt{x^2+1} = \log\left\{\sqrt{x^2+1}-x\right\}$$

Differentiating both sides w.r.t. x., we get

$$\frac{dy}{dx}\sqrt{x^2+1} + y\frac{1}{2\sqrt{x^2+1}}2x$$
$$= \frac{1}{\sqrt{x^2+1}-x}\times\left\{\frac{1}{2}\frac{2x}{\sqrt{x^2+1}}-1\right\}$$

$$\Rightarrow (x^2+1)\frac{dy}{dx} + xy = \sqrt{x^2+1}\frac{-1}{\sqrt{x^2+1}}$$

$$\Rightarrow (x^2+1)\frac{dy}{dx} + xy + 1 = 0$$

6 **(a)**

$$y = \tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}$$

$$= \tan^{-1}\left\{\sqrt{\frac{2\cos^2 x/2}{2\sin^2 x/2}}\right\}$$

$$= \tan^{-1}\left|\cot\frac{x}{2}\right| = \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right\} = \frac{\pi}{2}-\frac{x}{2}$$

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} = -\frac{1}{2}$$

7 **(d)**

Since, $\frac{dx}{dy} = \frac{1}{dy/dx} = \left(\frac{dy}{dx}\right)^{-1}$

$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dx}{dy}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

8 **(c)**

$f(x) = \sqrt{1-\sin 2x} = \sqrt{(\cos x - \sin x)^2}$

$= |\cos x - \sin x|$

$= \begin{cases} \cos x - \sin x, & \text{for } 0 \le x \le \pi/4 \\ -(\cos x - \sin x), & \text{for } \pi/4 < x \le \pi/2 \end{cases}$

$= \begin{cases} -(\cos x + \sin x), & \text{for } 0 < x < \pi/4 \\ \cos x + \sin x & \text{for } \pi/4 < x < \pi/2 \end{cases}$

9 **(b)**

$\frac{dy}{dx} = \frac{dy|dt}{dx|dt} = \frac{3t^2}{2t} = \frac{3}{2}\sqrt{x} \Rightarrow \frac{d^2y}{dx^2} = \frac{3}{4\sqrt{x}} = \frac{3}{4t}$

10 **(a)**

$y = x^2 + \frac{1}{y}$

$\Rightarrow y^2 = x^2y + 1$

$\Rightarrow 2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$

11 **(a)**

Let $g(x) = (\sin x)^{\text{In } x} = e^{\text{In } x.\text{In}(\sin x)}$

$f(x) = g'(x) = (\sin x)^{\text{In } x}\left[\cot x(\text{In } x) + \frac{\text{In}(\sin x)}{x}\right]$

Hence, $f\left(\frac{\pi}{2}\right) = g'\left(\frac{\pi}{2}\right) = 1(0+0) = 0$

12 **(c)**

We have $y^2 = P(x)$, where $P(x)$ is a polynomial of degree 3 and hence thrice differentiable,

Then $y^2 = P(x)$ (1)

Differentiate(1) w.r.t. x, we get

$2y\frac{dy}{dx} = P'(x)$ (2)

Again differentiate w.r.t. x, we get

$2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = P''(x)$

$\Rightarrow \frac{[P'(x)]^2}{2y^2} + 2y\frac{d^2y}{dx^2} = P''(x)$ [Using (2)]

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 2y^2 P''(x) - [P'(x)]^2$

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 2P(x)P''(x) - [P'(x)]^2$ [Using(1)]

$\Rightarrow 2y^3\frac{d^2y}{dx^2} = P(x)P''(x) - \frac{1}{2}[P'(x)]^2$

Again differentiating with respect to x, we get

$2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right) = P'''(x)P(x) + P''(x)$

$P'(x) - P'(x)P''(x)$

$= P'''(x)P(x)$

13 **(c)**

$y = \frac{\log \tan x}{\log \sin x}$

$\Rightarrow \frac{dy}{dx} = \frac{(\log \sin x)\left(\frac{\sec^2 x}{\tan x}\right) - (\log \tan x)(\cot x)}{(\log \sin x)^2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\pi/4} = \frac{-4}{\log 2}$ (On simplification)

14 **(c)**

$y = ae^{mx} + be^{-mx}$

$\Rightarrow \frac{dy}{dx} = ame^{mx} - mbe^{mx}$

Again $\frac{d^2y}{dx^2} = am^2e^{mx} + m^2be^{-mx}$

$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mx} + be^{-mx}) \Rightarrow \frac{d^2y}{dx^2} = m^2y$

$\Rightarrow \frac{d^2y}{dx^2} = m^2y = 0$

15 **(d)**

$y = \tan^{-1}\left(\frac{2^{x+1} - 2^x}{1 + 2^x.2^{x+1}}\right)$

$= \tan^{-1} 2^{(x+1)} - \tan^{-1} 2^x$

$\Rightarrow y' = \frac{2^{x+1}\text{ In } 2}{1 + (2^{x+1})^2} - \frac{2^x\text{In } 2}{1 + (2^x)^2}$

$\Rightarrow y'(0) = -\frac{1}{10}\text{In } 2$

16 **(a)**

$(\sin x)(\cos y) = \frac{1}{2}$

$\Rightarrow (\cos x)(\cos y) - \sin y \sin x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = (\cot x)(\cot y)$

$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x.\cot y - \operatorname{cosec}^2 y \cot x.\frac{dy}{dx}$

Now $\left(\frac{dy}{dx}\right)_{(\pi/4,\pi/4)} = 1$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{(\pi/4,\pi/4)} = -(2)(1) - (2)(1)(1) = -4$

17 **(d)**
$u + x^2 + y^2, x = s + 3t, y = 2s - t$

Now, $\frac{dx}{ds} = 1, \frac{dy}{ds} = 2 \quad (1)$

$\frac{d^2x}{ds^2} = 0, \frac{d^2y}{ds^2} = 0 \qquad (2)$

Now, $u = x^2 + y^2, \frac{du}{ds} = 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$

$\frac{d^2u}{ds^2} = 2\left(\frac{dx}{ds}\right)^2 + 2x\frac{d^2x}{ds^2} + 2\left(\frac{dy}{ds}\right)^2 + 2y\left(\frac{d^2y}{ds^2}\right)$

From (1) and (2), $\frac{d^2u}{ds^2} = 2 \times 1 + 0 + 2 \times 4 + 0 = 10$

18 **(a)**
$y = \sec(\tan^{-1} x)$

$= \sec\left(\sec^{-1}\sqrt{1+x^2}\right) = \sqrt{1+x^2}$

Differentiating w.r.t. x, we have $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{2}}$

19 **(b)**
$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\frac{1}{\sqrt{1-x^2}} - (\sin^{-1} x)\frac{1}{2}\frac{(-2x)}{\sqrt{1-x^2}}}{1-x^2}$

$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + x\left(\frac{\sin^{-1} x}{\sqrt{1-x^2}}\right) = 1 + xy$

20 **(c)**
$f'(x) = (kx + e^x)h'(x) + h(x)(k + e^x)$

$f'(0) = h'(0) + h(0)(k+1)$

$\Rightarrow 18 = -2 + 5(k+1) \Rightarrow k = 3$

21 **(c)**
$\frac{dy}{dx} = \frac{-\frac{1}{p^2}}{\frac{1}{p}} = -\frac{1}{p} = -y \Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

22 **(d)**
$\frac{d}{dx}\left(\tan^{-1}\frac{(\sqrt{x}\,(3-x))}{1-3x}\right)$

Put $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$\frac{d}{dx}\left(\tan^{-1}\frac{(\tan\theta(3-\tan^2\theta))}{1-3\tan^2\theta}\right)$

$\frac{d}{dx}\left(\tan^{-1}\frac{(3\tan\theta - \tan^3\theta)}{1-3\tan^2\theta}\right)$

$\frac{d}{dx}[\tan^{-1}(\tan 3\theta)] = \frac{d}{dx}(3\theta)$

$= \frac{d}{dx}\left[3\tan^{-1}\sqrt{x}\right] = \frac{3}{2\sqrt{x}\,(1+x)}$

23 **(c)**
$y = \sin x + e^x \Rightarrow \frac{dy}{dx} = \cos x + e^x$

$\Rightarrow \frac{dx}{dy} = (\cos x + e^x)^{-1} \quad (1)$

Again, $\frac{d^2x}{dy^2} = -(\cos x + e^x)^{-2}(-\sin x + e^x)\frac{dx}{dy}$

Substituting the value of $\frac{dy}{dx}$ from (1)

$\frac{d^2x}{dy^2} = \frac{(\sin x - e^x)}{(\cos x + e^x)^2}(\cos x + e^x)^{-1}$

$= \frac{\sin x - e^x}{(\cos x + e^x)^3}$

24 **(d)**
We have $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \sin(\log a)$

$\Rightarrow \frac{1-\tan^2\theta}{1+\tan^2\theta} = \sin(\log a)$, on putting $y = x\tan\theta$

$\Rightarrow \cos 2\theta = \sin(\log a)$

$\Rightarrow 2\theta = \cos^{-1}(\sin(\log a))$

$\Rightarrow \theta = \frac{1}{2}\cos^{-1}(\sin(\log a))$

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\cos^{-1}(\sin(\log a))$

$\Rightarrow \frac{y}{x} = \tan\left(\frac{1}{2}\cos^{-1}(\sin(\log a))\right)$

Differentiating w.r.t. x

$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = 0$

$\Rightarrow x\frac{dy}{dx} - y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

25 **(b)**

$y = \frac{(a-x)^{3/2} + (x-b)^{3/2}}{\sqrt{a-x} + \sqrt{x-b}}$

$= \frac{(\sqrt{a-x}+\sqrt{x-b})\left(a-x-\sqrt{a-x}\sqrt{x-b}+x-b\right)}{\sqrt{a-x}+\sqrt{x-b}}$

$= a - b - \sqrt{a-x}\sqrt{x-b}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a-x}}\sqrt{x-6} - \frac{1}{2\sqrt{x-b}}\sqrt{a-x}$

$= \frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

26 **(a)**

In the neighbourhood of $x = 7\pi/6$, we have
$f(x) = |\sin x + \cos x| = -\sin x - \cos x$

$\Rightarrow f'(x) = -\cos x + \sin x \Rightarrow f'(7\pi/6) = -\cos(7\pi/6) + \sin(7\pi/6) = \frac{\sqrt{3}-1}{2}$

27 **(c)**

$y'(x)$
$= f'\left(f\left(f(f(x))\right)\right)f'\left(f(f(x))\right)f'(f(x))f'(x)$

$\Rightarrow y'(0)$
$= f'(f(f(f(0))))\,f'(f(f(0)))f'(f(0))f'(0)$

$= f'\left(f(f(0))\right)f'(f(0))f'(0)f'(0)$

$= f'(f(0))f'(0)f'(0)f'(0)$

$= f'(0)f'(0)f'(0)f'(0)$

$= \left(f'(0)\right)^4 = 2^4 = 16$

28 **(a)**

$y = (\sin x)^{\tan x}$
$\Rightarrow \log y = \tan x \log \sin x$
Differentiating w.r.t. x, we get
$\frac{1}{y}\frac{dy}{dx} = \sec^2 x \log \sin x + \tan x \frac{1}{\sin x}.\cos x$
$\Rightarrow \frac{dy}{dx} = (\sin x)^{\tan x}[1 + \sec^2 x \log \sin x]$

29 **(a)**

$f(10) = \sin^{-1}\cos 10 = \sin^{-1}\sin\left(\frac{\pi}{2} - 10\right)$

$= -\sin^{-1}\sin\left(10 - \frac{\pi}{2}\right)$

$= -\sin^{-1}\sin\left(3\pi - 10 + \frac{\pi}{2}\right) = -\left(3\pi + \frac{\pi}{2} - 10\right) = 10 - \frac{7\pi}{2}$

$f'(x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}} = \frac{-\sin x}{|\sin x|} \Rightarrow f'(10) = \frac{-\sin 10}{|\sin 10|} = 1$

So, $f(10) + f'(10) = 11 - \frac{7\pi}{2}$

30 **(c)**

$f(x) = xe^x$

$f'(x) = e^x + xe^x$

$f''(x) = e^x + e^x + xe^x$

$f'''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x$

…

…

$f^n(x) = ne^x + xe^x$

Now, $f^n(x) = 0$

$\Rightarrow ne^x + xe^x = 0 \Rightarrow x = -n$

31 **(a)**

$f(x) = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right] + x^x$

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}} + x^x$

$\Rightarrow f'(x) = -\frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{1+x}} + x^x(1 + \log x)$

$\Rightarrow f'(1) = -\frac{1}{4} + 1 = \frac{3}{4}$

32 **(b)**

$y = ax^{n+1} + bx^{-n}$

$\Rightarrow \frac{dy}{dx} = (n+1)ax^n - nbx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = n(n+1)ax^{n-1} + n(n+1)bx^{-n-2}$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)y$

33 **(a)**

Given $f = f' + f'' + f''' + \cdots \infty$

$\Rightarrow f' = f'' + f''' + f'''' + \cdots \infty$

$\Rightarrow f - f' = f'$

$\Rightarrow f = 2f'$

Hence, $\frac{f'}{f} = 1/2 \Rightarrow \int \frac{f'}{f} dx = \int \frac{1}{2} dx$

$\Rightarrow \log f(x) = x/2 + c$

$\Rightarrow f(x) = e^{x/2+c}$

Also, $f(0) = 1 \Rightarrow c = 0 \Rightarrow f(x) = e^{x/2}$

34 **(c)**

$\lim_{x\to 0} \frac{g(x)f(a) - g(a)f(x)}{x-a}$

$= \lim_{x\to 0} \frac{g(a+h)f(a) - g(a)f(a) + g(a)f(a) - g(a)f(a+h)}{h}$

$= \lim_{x\to 0} f(a)\left[\frac{g(a+h) - g(a)}{h}\right] - \lim_{x\to 0} g(a)\left[\frac{f(a+h) - f(a)}{h}\right]$

$= f(a)g'(a) - g(a)f'(a)$

$= 2 \times 2 - (-1) \times 1 = 5$

35 **(b)**

$f(x) = \sqrt{1 + \cos^2(x)^2}$

$\Rightarrow f'(x)$

$= \frac{1}{2\sqrt{1+\cos^2(x)^2}}(2\cos x^2)(-\sin x^2)(2x)$

$\Rightarrow f'(x) = \frac{-x \sin 2x^2}{\sqrt{1 + \cos^2(x^2\ )}}$

$\Rightarrow f'\left(\frac{\sqrt{\pi}}{2}\right) = \frac{-\frac{\sqrt{\pi}}{2}\sin\frac{2\pi}{4}}{\sqrt{1+\cos^2\frac{\pi}{4}}} = \frac{-\frac{\sqrt{\pi}}{2}1}{\sqrt{\frac{3}{2}}}$

$\therefore f'\left(\frac{\sqrt{\pi}}{2}\right) = -\sqrt{\frac{\pi}{6}}$

36 **(a)**

We have

$y^{1/m} = (x + \sqrt{1+x^2})$

$\Rightarrow y = \left(x + \sqrt{1+x^2}\right)^m$

$\Rightarrow \frac{dy}{dx} = m\left(x + \sqrt{1+x^2}\right)^{m-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= m\frac{\left(x+\sqrt{1+x^2}\right)^m}{\sqrt{1+x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{my}{\sqrt{1+x^2}}$

$\Rightarrow y_1^2(1+x^2) = m^2y^2$

$\Rightarrow 2y_1y_2(1+x^2) + 2xy_1^2 = 2m^2yy_1$

$\Rightarrow y_2(1+x^2) + xy_1 = m^2y$

37 **(b)**

$2xf'(x^2) = 3x^2 \Rightarrow 4f'(2) = 12 \Rightarrow f'(4) = 3$

38 **(c)**

$y = x^{(x^x)}$

$\Rightarrow \log y = x^x \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dz}{dx}\log x + \frac{1}{x}z$ (where $x^x = z$)

$\Rightarrow \frac{dy}{dx} = x^{(x^x)}[x^x(\log ex)\log x + x^{x-1}]$

$\left(\because \frac{dz}{dx} = x^x \log ex\right)$

39 **(a)**
$\frac{d}{dx}\cos^{-1}\sqrt{\cos x} = \frac{\sin x}{2\sqrt{\cos x}\sqrt{1-\cos x}}$

$= \frac{\sqrt{1-\cos^2 x}}{2\sqrt{\cos x}\sqrt{1-\cos x}} = \frac{1}{2}\sqrt{\frac{1+\cos x}{\cos x}}$

40 **(d)**
$y = a\sin x + b\cos x$

Differentiating with respect to x, we get

$\frac{dy}{dx} = a\cos x - b\sin x$

Now, $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$

$= a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$ and

$y^2 = (a\sin x + b\cos x)^2$

$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$

So, $\left(\frac{dy}{dx}\right)^2 + y^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) =$ constant

41 **(b)**
$D = \begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$

$= p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$

$= -p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ \cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix} = 0$

42 **(a)**
Given that $g^{-1}(x) = f(x) \Rightarrow x = g(f(x))$ or $g'(f(x))f'(x) = 1$

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

$\Rightarrow g''(f(x))f'(x) = \frac{-f''(x)}{[f'(x)]^2} \Rightarrow g''(f(x)) = \frac{-f''(x)}{[f'(x)]^3}$

43 **(d)**
Let $y = \log x$

$\Rightarrow y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3}, \dots, y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$

44 **(b)**
$f(x) = |x^2 - 5x + 6|$

$= \begin{cases} x^2 - 5x + 6 & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2 < x < 3 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} (2x - 5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x - 5) & \text{if } 2 < x < 3 \end{cases}$

45 **(a)**
$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{f'(x^3)3x^2}{g'(x^2)2x} = \frac{\cos x^3 3x^2}{\sin x^2 2x}$

$= \frac{3}{2}x\cos x^3 \operatorname{cosec} x^2$

46 **(d)**
Let $u = y^2$ and $v = x^2$

$\therefore \frac{du}{dx} = \frac{d}{dx}y^2 = \left(\frac{d}{dy}y^2\right)\left(\frac{dy}{dx}\right)$

$= 2y(1-2x) = 2(x - x^2)(1-2x) = 2x(1-x)(1-2x)$ (1)

and $\frac{dv}{dx} = 2x$ (2)

Hence, $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{2x(1-x)(1-2x)}{2x}$ (from(1) and (2))

$= (1-x)(1-2x) = 1 - 3x + 2x^2$

47 **(b)**
$\sqrt{x} = \cos\theta$

$x \in \left(0, \frac{1}{2}\right) \Rightarrow \sqrt{x} = \cos\theta \in \left(0, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\Rightarrow 2\theta \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow f(x) = 2\sin^{-1}\sqrt{1-\cos^2\theta} + \sin^{-1}(2\sqrt{\cos^2\theta\sin^2\theta})$

$= 2\sin^{-1}(\sin\theta) + \sin^{-1}(2\sin\theta\cos\theta)$

$= 2\theta + \sin^{-1}(\sin 2\theta)$

$= 2\theta + \pi - 2\theta$

$= \pi$

$\Rightarrow f'(x) = 0$

48 **(b)**

Given $f\left(\frac{5x-3y}{2}\right) = \frac{5f(x)-3f(y)}{2}$

$\Rightarrow f\left(\frac{5x-3y}{5-3}\right) = \frac{5f(x)-3f(y)}{5-3}$, which satisfies section formula for abscissa on L.H.S. and ordinate on R.H.S. Hence, $f(x)$ must be the linear function (as only straight line satisfies such section formula)

But $f(0) = 3 \Rightarrow b = 3, f'(0) = 2 \Rightarrow a = 2$

Thus, $f(x) = 2x + 3 \Rightarrow$ Period of $\sin(f(x)) = \sin(2x+3)$ is π

49 **(b)**

$f(g(x)) = x$

$\Rightarrow f'(g(x))g'(x) = 1$

$\Rightarrow (e^{g(x)} + 1)g'(x) = 1$

$\Rightarrow (e^{g(f(\log 2))} + 1)g'(f(\log 2)) = 1$

$\Rightarrow (e^{\log 2} + 1)g'(f(\log 2)) = 1$

$\Rightarrow g'(f(\log 2)) = 1/3$

50 **(b)**

Let $y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\Rightarrow y = \sin^2\cot^{-1}\left\{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\}$

$= \sin^2\cot^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow y = \sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$= \cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2} = \frac{1+x}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$

51 **(a)**

$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$= \cot^{-1}\left[\frac{2+2\cos x}{2\sin x}\right] = \cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$

$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

52 **(a)**

Let $\cos a = \frac{5}{13},$ then $\sin a = \frac{12}{13}$

So, $y = \cos^{-1}(\cos a.\cos x - \sin a.\sin x)$

$\Rightarrow y = \cos^{-1}\{\cos(x+a)\} = x + a$ ($\because x + a$ is in the first or the second quadrant)

$\Rightarrow \frac{dy}{dx} = 1$

53 **(a)**

Let $x = \sec\theta$

Then $y = \tan^{-1}\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$= \tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \tan^{-1}\left(\cot\frac{\theta}{2}\right)$

$\Rightarrow y = \tan^{-1}\left\{\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right\} = \frac{\pi}{2} - \frac{1}{2}\sec^{-1}x$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2} \times \frac{1}{|x|\sqrt{x^2-1}}$

54 **(b)**

$f(x) = 1 + x^2 + x^4 + x^6 + \cdots \infty$, where $|x| \le 1$

$\Rightarrow f^n(0) = n!$, where n is even

55 **(c)**

We have $y = \sqrt{\frac{1-x}{1+x}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{1/2-1} \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$= -\sqrt{\frac{1+x}{1-x}}\frac{1}{(1+x)^2}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = -\sqrt{\frac{1+x}{1-x}}\frac{1}{(1+x)^2}(1-x^2)$$

$$\Rightarrow (1-x)^2\frac{dy}{dx} = -\sqrt{\frac{1-x}{1+x}}$$

$$\Rightarrow (1-x)^2\frac{dy}{dx} = -y$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} + y = 0$$

56 **(c)**

We have $y = \tan^{-1}\left(\frac{\log e - \log x^2}{\log e + \log x^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$$= \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$$

$$= \tan^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3 + \tan^{-1}(2\log x)$$

$$= \tan^{-1}1 + \tan^{-1}3$$

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$$

57 **(c)**

$f(x) = x + \tan x$

$f\left(f^{-1}(y)\right) = f^{-1}(y) + \tan f^{-1}(y)$

$y = g(y) + \tan g(y)$

$x = g(x) + \tan g(x)$

Differentiating, we get $1 = g'(x) + \sec^2 g(x) g'(x)$

$$\Rightarrow g'(x) = \frac{1}{1+\sec^2 g(x)} = \frac{1}{2+[g(x)-x]^2}$$

58 **(a)**

Since g is the inverse function of f, we have $f\{g(x)\} = x$

$$\Rightarrow \frac{d}{dx}(f\{g(x)\}) = 1$$

$$\Rightarrow f'\{g(x)\}.g'(x) = 1$$

$$\Rightarrow \sin\{g(x)\}\, g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{\sin\{g(x)\}}$$

59 **(b)**

For $x > 1$, we have $f(x) = |\log|x|| = \log x$

$$\Rightarrow f'(x) = \frac{1}{x}$$

For $x < -1$, we have $f(x) = |\log|x|| = \log(-x)$

$$\Rightarrow f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have $f(x) = |\log|x|| = -\log x$

$$\Rightarrow f'(x) = \frac{-1}{x}$$

For $-1 < x < 0$, we have $f(x) = -\log(-x)$

$$\Rightarrow f'(x) = \frac{-1}{x}$$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

60 **(b)**

$$y = x + e^x \Rightarrow \frac{dy}{dx} = 1 + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{1+e^x}\right) \Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{1}{1+e^x}\right)\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^2}\frac{1}{(1+e^x)} = -\frac{e^x}{(1+e^x)^3}$$

61 **(c)**

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

$$\Rightarrow \frac{dy}{dx} = 0 + 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{n-1}}{(n-1)!}$$

$$\Rightarrow \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$$

$$\Rightarrow \frac{dy}{dx} = y - \frac{x^n}{n!}$$

62 **(b)**

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, and $z = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Putting $x = \tan\theta$ in y we get

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Putting $x = \sin\theta$ in z, we get

$$z = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right)$$
$$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\sin^{-1}x$$

$$\Rightarrow \frac{dz}{dx} = \frac{2}{\sqrt{1-x^2}}$$

Thus, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{1}{4(1+x^2)}\sqrt{1-x^2} \Rightarrow \left(\frac{dy}{dz}\right)_{x=0} = \frac{1}{4}$

63 **(c)**

In neighbourhood of $x = \frac{2\pi}{3}$, $|\cos x| = -\cos x$ and $|\sin x|$

$= \sin x$

$\Rightarrow y = -\cos x + \sin x$

$$\Rightarrow \frac{dy}{dx} = \sin x + \cos x$$

$\Rightarrow$ At $x = \frac{2\pi}{3}, \frac{dy}{dx} = \sin\frac{2\pi}{3} + \cos\frac{2\pi}{3} = \frac{\sqrt{3}-1}{2}$

64 **(b)**

Let $f(x) = ax^2 + bx + c$

As given that $f(x) > 0, \forall\, x \in R$

$\therefore a > 0$ and $b^2 - 4ac < 0$ (1)

Now, $g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a+b)x + (2a+b+c)$

Here $D = (2a+b)^2 - 4a(2a+b+c)$

$= 4a^2 + b^2 + 4ab - 8a^2 - 4ab - 4ac$

$= -4a^2 + (b^2 - 4ac) < 0$

Also $a > 0$ from(1), $\Rightarrow g(x) > 0, \forall x \in R$

65 **(c)**

$(a^2 - 2a - 15)e^{ax} + (b^2 - 2b - 15)d^{bx} = 0$

$\Rightarrow (a^2 - 2a - 15) = 0$ and $b^2 - 2b - 15 = 0$

$\Rightarrow (a-5)(a+3) = 0$ and $(b-5)(b+3) = 0$

$\Rightarrow a = 5$ or -3 and $b = 5$ or-3

$\therefore a \neq b$ hence $a = 5$ and $b = -3$

Or $a = -3$ and $b = 5$

$\Rightarrow ab = -15$

66 **(b)**

We have $x = \emptyset(t), y = \psi(t)$. Therefore,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\psi'}{\phi'}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{\psi'}{\phi'}\right) = \frac{d}{dt}\left(\frac{\psi'}{\phi'}\right)\frac{dt}{dx}$$

$$= \frac{\phi'\psi'' - \psi'\phi''}{\phi'^2}\frac{1}{\phi'} = \frac{\phi'\psi'' - \psi'\phi''}{\phi'^3}$$

67 **(a)**

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x + \sqrt{x^2+a^2}\right)^n\right]$$

$$= n\left(x + \sqrt{x^2+a^2}\right)^{n-1} . \frac{d}{dx}\left(x + \sqrt{x^2+a^2}\right)$$

$$= n\left(x + \sqrt{x^2+a^2}\right)^{n-1}\left(\frac{\sqrt{x^2+a^2} + a^2}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{n\left(x + \sqrt{x^2+a^2}\right)^n}{\sqrt{x^2+a^2}}$$

$$= \frac{ny}{\sqrt{x^2+a^2}}$$

68 **(a)**

Since, $f(x) = e^{g(x)}$

$\Rightarrow \; e^{g(x+1)} = f(x+1) = xf(x) = xe^{g(x)}$

and $\quad g(x+1) = \log x + g(x)$

$\Rightarrow \; g(x+1) - g(x) = \log x$ ...(i)

Replacing x by $x - \frac{1}{2}$, we get

$$g\left(x + \frac{1}{2}\right) - g\left(x - \frac{1}{2}\right) = \log(x - \frac{1}{2})$$

$= \log(2x-1) - \log 2$

$\therefore \; g''\left(x + \frac{1}{2}\right) - g''\left(x - \frac{1}{2}\right) = -\frac{4}{(2x-1)^2}$ ...(ii)

On substituting, $x = 1,2,3, \dots N$ in Eq. (ii) and adding, we get

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$$

$$= -4\left\{1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2}\right\}$$

69 **(a)**

$y = 2 \text{ In } (1 + \cos x)$

$$\frac{dy}{dx} = \frac{-2\sin x}{1 + \cos x}$$

$$\frac{d^2y}{dx^2} = -2\left[\frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}\right]$$

$$= -2\left[\frac{\cos x + 1}{(1+\cos x)^2}\right] = \frac{-2}{(1+\cos x)}$$

Now $2e^{-y/2} = 2 \cdot e^{-\frac{\text{In }(1+\cos x)^2}{2}} = \frac{2}{(1+\cos x)}$

$$\therefore \frac{d^2y}{dx^2} + \frac{2}{e^{y/2}} = 0$$

70 **(a)**

$u(x) = 7v(x) \Rightarrow u'(x) = 7v'(x) \Rightarrow p = 7$ (given)

1. Again $\frac{u(x)}{v(x)} = 7 \Rightarrow \left(\frac{u(x)}{v(x)}\right) = 0 \Rightarrow q = 0$

2. Now $\frac{p+q}{p-q} = \frac{7+0}{7-0} = 1$

71 **(b)**

Given, $f''(x) = -f(x)$

$\Rightarrow g'(x) = -f(x)$ and $f'(x) = g(x)$ ...(i)

Now, $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$

$\therefore F'(x) = 2\left(f\left(\frac{x}{2}\right)\right).f'\left(\frac{x}{2}\right).\frac{1}{2}$

$+2\left(g\left(\frac{x}{2}\right)\right).g'\left(\frac{x}{2}\right).\frac{1}{2} = 0$

[using Eq.(i)]

$\therefore \; F(x)$ is a constant $\Rightarrow F(10) = F(5) = 5$

72 **(b)**

$$\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{\cos t - t\sin t}{1 + \cos t}$$

$$\therefore \; \frac{d^2x}{dy^2} = \frac{\frac{d}{dt}\left(\frac{dx}{dy}\right)}{\frac{dy}{dt}}$$

$$= \frac{\frac{(-2\sin t - t\cos t)(1+\cos t) - (\cos t - t\sin t)(-\sin t)}{(1+\cos t)^2}}{1 + \cos t}$$

Now, put $t = \pi/2$

73 **(a)**

$y = f(x) - f(2x) \Rightarrow y' = f'(x) - 2f'(2x)$

$\Rightarrow y'(1) = f'(1) - 2f'(2) = 5$, and (1)

$y'(2) = f'(2) - 2f'(4) = 7$ (2)

Now let $y = f(x) - f(4x)$

$\Rightarrow y' = f'(x) - 4f'(4x)$

$\Rightarrow y'(1) = f'(1) - 4f'(4)$ (3)

Substituting the value of $f'(2) = 7 + 2f'(4)$ in(1), we get

$f'(1) - 2(7 + 2f'(4)) = 5$

$f'(1) - 4f'(4) = 19$

74 **(a)**

$$y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$$

$$\Rightarrow y = \frac{\left(\sqrt{a+x} - \sqrt{a-x}\right)^2}{(a+x) - (a-x)}$$

$$\Rightarrow y = \frac{(a+x) + (a-x) - 2\left(\sqrt{a^2-x^2}\right)}{2x}$$

$$= \frac{2a - 2\sqrt{a^2-x^2}}{2x} = \frac{a - \sqrt{a^2-x^2}}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{x\left[-\frac{1}{2\sqrt{a^2-x^2}}(-2x)\right]-\left(a-\sqrt{a^2-x^2}\right)}{x^2}$$

$$=\frac{x^2-a\sqrt{a^2-x^2}+a^2-x^2}{x^2\sqrt{a^2-x^2}}=\frac{a\left(a-\sqrt{a^2-x^2}\right)}{x^2\sqrt{a^2-x^2}}$$

$$=\frac{a}{x\sqrt{a^2-x^2}}\left[\frac{a-\sqrt{a^2-x^2}}{x}\right]=\frac{ay}{x\sqrt{a^2-x^2}}\ \text{[by(1)]}$$

75 **(b)**
$y=2\cos x\cos 3x=\cos 4x+\cos 2x$

$$\Rightarrow\frac{d^{20}y}{dx^{20}}=4^{20}\cos 4x+2^{20}\cos 2x$$

76 **(c)**
Here, $y=t^{10}+1$ and $x=t^8+1$

$\therefore t^8=x-1\Rightarrow t^2=(x-1)^{1/4}$

So, $y=(x-1)^{5/4}+1$

Differentiate both sides w.r.t. x, we get $\frac{dy}{dx}=\frac{5}{4}\,(x-1)^{1/4}$

Again, differentiate both sides w.r.t., we get

$$\frac{d^2y}{dx^2}=\frac{5}{16}(x-1)^{-3/4}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{5}{16(x-1)^{3/4}}=\frac{5}{16(t^2)^3}=\frac{5}{16t^6}$$

77 **(a)**
As $f(x)=x^4\tan(x^3)-x\ \text{In}\ (1+x^2)$ is odd, $\Rightarrow \frac{d^3f(x)}{dx^3}$ is even

$\Rightarrow\frac{d^4f(x)}{dx^4}=0$ at $x=0$

78 **(c)**

$$f(x)=e^x-e^{-x}-2\sin x-\frac{2}{3}x^3$$

$$f^{\text{I}}(x)=e^x+e^{-x}-2\cos x-2x^2$$

$$f^{\text{II}}(x)=e^x-e^{-x}+2\sin x-4x$$

$$f^{\text{III}}(x)=e^x+e^{-x}+2\cos x-4$$

$$f^{\text{IV}}(x)=e^x-e^{-x}-2\sin x$$

$$f^{\text{V}}(x)=e^x+e^{-x}-2\cos x$$

$$f^{\text{VI}}(x)=e^x-e^{-x}+2\sin x$$

$$f^{\text{VII}}(x)=e^x+e^{-x}+2\cos x$$

Clearly, $f^{\text{VII}}(0)$ is non-zero

79 **(b)**
Since $g(x)$ is the inverse of function $f(x)$, therefore $gof(x)=I(x)$ for all x

Now $gof(x)=I(x),\forall x$

$\Rightarrow(gof)'(x)=1,\forall x$

$\Rightarrow g'(f(x))f'(x)=1,\forall x$

$$\Rightarrow g'(f(x))=\frac{1}{f'(x)},\forall x$$

$\Rightarrow g'(f(c))=\frac{1}{f'(c)}$ (putting $x=c$)

80 **(b)**

$$\frac{dy}{dx}=-[(2-x)(3-x)\dots(n-x)+(1-x)(3-x)\dots(n-x)+\cdots(1-x)(2-x)\dots(n-1-x)]$$

At $x=1$

$$\frac{dy}{dx}=-[(n-1)!+0+\cdots+0]=(-1)(n-1)!$$

81 **(a)**
$ax^2+2hxy+by^2=1$

Differentiating both sides w.r.t. x, we get

$$2ax+2hx\frac{dy}{dx}+2hy+2by\frac{dy}{dx}=0$$

$$\Rightarrow\frac{dy}{dx}=-\frac{ax+hy}{hx+by}$$

Again differentiating w.r.t. x, we get

$$\Rightarrow\frac{d^2y}{dx^2}$$

$$=-\left[\frac{(hx+by)\left(a+h\frac{dy}{dx}\right)-(ax+hy)\left(h+b\frac{dy}{dx}\right)}{(hx+by)^2}\right]$$

$$= -\frac{\left[y(ab-h^2)+\frac{dy}{dx}(h^2x-abx)\right]}{(hx+by)^2}$$

$$= \frac{(h^2-ab)\left(y-x\frac{dy}{dx}\right)}{(hx+by)^2}$$

$$= \frac{(h^2-ab)}{(hx+by)^2}\left[y+x\frac{ax+hy}{hx+by}\right]$$

$$= \frac{h^2-ab}{(hx+by)^2}$$

82 **(c)**

$y=\sqrt{\log x+y}$

$\Rightarrow y^2=\log x+y$

$\Rightarrow 2y\,\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{x(2y-1)}$

83 **(b)**

From the given relation $\frac{y}{x}=\log x-\log(a+bx)$

Differentiating w.r.t. x, we get $\frac{\left(x\frac{dy}{dx}\right)-y}{x^2}=\frac{1}{x}-\frac{b}{a+bx}=\frac{a}{x(a+bx)}$

$\therefore x\frac{dy}{dx}-y=\frac{ax}{a+bx}$ (1)

Differentiating again w.r.t. x, we get

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-axb}{(a+bx)^2}$

$\Rightarrow x\frac{d^2y}{dx^2}=\frac{a^2}{(a+bx)^2}$

$\Rightarrow x^3\frac{d^2y}{dx^2}=\frac{a^2x^2}{(a+bx)^2}=\left(x\frac{dy}{dx}-y\right)^2$ [by(1)]

84 **(a)**

$y=f(x)$ is an even function and $y=g(x)$ is an odd function

$\Rightarrow h(x)=f(x)g(x)$ is an odd function

$\Rightarrow h(x)=-h(-x)$

$\Rightarrow h'(x)=h'(-x)$

$\Rightarrow h''(x)=-h''(-x)$

$\Rightarrow h'''(x)=h'''(-x)$

Now, we cannot determine the value of $h'''(0)$

85 **(b)**

Putting $x=\sin\theta$ and $y=\sin\emptyset$

$\cos\theta+\cos\emptyset=a(\sin\theta-\sin\emptyset)$

$\Rightarrow 2\cos\frac{\theta+\emptyset}{2}\cos\frac{\theta-\emptyset}{2}=a\left(2\cos\frac{\theta+\emptyset}{2}\sin\frac{\theta-\emptyset}{2}\right)$

$\Rightarrow\frac{\theta-\emptyset}{2}=\cot^{-1}a$

$\Rightarrow\sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

$\Rightarrow\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

86 **(a)**

$y^2=ax^2+bx+c$

$\Rightarrow 2y\frac{dy}{dx}=2ax+b$

$\Rightarrow 2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}=2a$

$\Rightarrow y\frac{d^2y}{dx^2}=a-\left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\frac{d^2y}{dx^2}=a-\left(\frac{2ax+b}{2y}\right)^2$

$\Rightarrow y\frac{d^2y}{dx^2}=\frac{4ay^2-(2ax+b)^2}{4y^2}$

$\Rightarrow 4y^3\frac{d^2y}{dx^2}=4a(ax^2+bx+c)-(4a^2x^2+4abx+b^2)$

$\Rightarrow 4y^3\frac{d^2y}{dx^2}=4ac-b^2$ =Constant

87 **(a)**

$y=f(x^2)\Rightarrow\frac{dy}{dx}=f'(x^2)2x=2x\sqrt{2(x^2)^2-1}$

At $x=1,\frac{dy}{dx}=2\times1\times\sqrt{2-1}=2$

88 **(b)**

We have $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$

$\Rightarrow (x^2+y^2)^2 = t^2 + \frac{1}{t^2} - 2$

$\Rightarrow (x^2+y^2)^2 = x^4 + y^4 - 2$

$\Rightarrow 2x^2y^2 = -2$

$\Rightarrow x^2y^2 = -1$

$\Rightarrow y^2 = -\frac{1}{x^2}$

$\Rightarrow 2y\frac{dy}{dx} = \frac{2}{x^3}$

$\Rightarrow x^3y\frac{dy}{dx} = 1$

89 **(a)**

$F'(x) = \left[f\left(\frac{x}{2}\right).f'\left(\frac{x}{2}\right) + g\left(\frac{x}{2}\right)g'\left(\frac{x}{2}\right)\right]$

Here $g(x) = f'(x)$

and $g'(x) = f''(x) = -f(x)$

So $F'(x) = f\left(\frac{x}{2}\right)g\left(\frac{x}{2}\right) - f\left(\frac{x}{2}\right)g\left(\frac{x}{2}\right) = 0$

$\Rightarrow F(x)$ is a constant function

$\Rightarrow F(10) = 5$

90 **(b)**

$y = \cos^{-1}(\cos x) = \cos^{-1}\{\cos[2\pi - (2\pi - x)]\}$

$= \cos^{-1}[\cos(2\pi - x)]$

$= 2\pi - x$

$\therefore \frac{dy}{dx} = -1$ at $x = \frac{5\pi}{4}$

91 **(d)**

Given $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is constant

$\Rightarrow f'''(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow f'''(x)]_{x=0} = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0 (\because R_1 \equiv R_2)$

$=$ Independent of p

92 **(a,c)**

$\frac{d}{dx}\{f_n(x)\} = \frac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= e^{f_{n-1}(x)}\frac{d}{dx}\{f_{n-1}(x)\} = f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$

$= f_n(x).\frac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= f_n(x).e^{f_{n-2}(x)}\frac{d}{dx}\{f_{n-2}(x)\}$

$= f_n(x)f_{n-1}(x)\frac{d}{dx}\{f_{n-2}(x)\}$

...

$= f_n(x)f_{n-1}(x)\ldots f_2(x)\frac{d}{dx}\{f_1(x)\}$

$= f_n(x).f_{n-1}(x)\ldots f_2(x)\frac{d}{dx}\{e^{f_0(x)}\}$

$= f_n(x).f_{n-1}(x)\ldots f_2(x)\{e^{f_0(x)}\}\frac{d}{dx}\{f_0(x)\}$

Use $e^{f_0(x)} = f_1(x)$ and $f_0(x) = x$

93 **(b,c)**

$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}}\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = -\frac{1+x^2}{\sqrt{(1-x^2)^2}}\frac{2(1+x^2)-4x^2}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = -2\frac{(1+x^2)}{|1-x^2|}\frac{1-x^2}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = -2\left(\frac{1-x^2}{|1-x^2|}\right)\left(\frac{1}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2}, & \text{if } |x| > 1 \\ \frac{-2}{1+x^2}, & \text{if } |x| < 1 \end{cases}$

94 **(a,c)**

$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{e^{-\sqrt{x}}}{2\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$

$= \frac{\sqrt{\left(e^{\sqrt{x}} + e^{-\sqrt{x}}\right)^2 - 4}}{2\sqrt{x}} = \frac{\sqrt{y^2-4}}{2\sqrt{x}}$

95 **(a,b,c)**

$f(x) = |x^2 - 3|x| + 2|$

$$= \begin{cases} |x^2 - 3x + 2|, & x \geq 0 \\ |x^2 + 3x + 2|, & x < 0 \end{cases}$$

$$= \begin{cases} x^2 - 3x + 2, & x^2 - 3x + 2 \geq 0, \quad x \geq 0 \\ -x^2 + 3x + 2, & x^2 - 3x + 2 < 0 \quad x \geq 0 \\ x^2 + 3x + 2, & x^2 + 3x + 2 \geq 0 \quad x < 0 \\ -x^2 - 3x + 2, & x^2 + 3x + 2 < 0 \quad x < 0 \end{cases}$$

$$= \begin{cases} x^2 - 3x + 2, & x \in [0,1] \cup [2, \infty) \\ -x^2 + 3x - 2, & x \in (1, 2) \\ x^2 + 3x + 2, & x \in (-\infty, -2] \cup [-1, 0) \\ -x^2 - 3x + 2 & x \in (-2, -1) \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2x - 3, & x \epsilon (0,1) \cup (2, \infty) \\ -2x + 3, & x \in (1,2) \\ 2x + 3, & x \in (-\infty, -2) \cup (-1, 0) \\ -2x - 3, & x \in (-2, -1) \end{cases}$$

96 **(b,c)**
$y = \sin^{-1}(\sin x)$
$= x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
$= \pi - x, \frac{\pi}{2} < x \leq \frac{3\pi}{2}$
$\therefore \frac{dy}{dx} = 1, -\frac{\pi}{2} < x < \frac{\pi}{2},$
$= -1, \frac{\pi}{2} < x < \frac{3\pi}{2}$

97 **(b,c)**
$$y = \frac{(x^2 + 1)^2 - 3x^2}{x^2 + \sqrt{3}x + 1}$$
$$= \frac{(x^2 + 1 + \sqrt{3}x)(x^2 + 1 - \sqrt{3}x)}{x^2 + 1 + \sqrt{3}x}$$

$\frac{dy}{dx} = 2x - \sqrt{3} \Rightarrow a = 2$ and $b = -\sqrt{3}$

$a - b = 2 + \sqrt{3} = \tan\frac{5\pi}{12} = \cot\frac{\pi}{12}$

98 **(b,c,d)**
1 is a root of $f(x) = 0, f'(x) = 0$ and $f''(x) = 0$, or

1 is a root of $ax^3 + bx^2 + bx + d = 0 \quad (1)$

$3ax^2 + 2bx + b = 0 \quad (2)$

$\Rightarrow a + 2b + d = 0$

$a + b = 0$

$\Rightarrow b + d = 0$ and $a = d$

99 **(a)**
$u = \sin^{-1} x$
$v = \sin^{-1}(3x - 4x^3)$
$$= \begin{cases} -\pi - 3\sin^{-1} x, & -1 \leq x \leq -\frac{1}{2} \\ 3\sin^{-1} x, & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ \pi - 3\sin^{-1} x, & \frac{1}{2} \leq x \leq 1 \end{cases}$$
$$\therefore \frac{du}{dv} = \begin{cases} -\frac{1}{3}, & -1 \leq x \leq -\frac{1}{2} \\ \frac{1}{3}, & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ -\frac{1}{3}, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

100 **(a,c)**
$x^3 - 2x^2y^2 + 5x + y - 5 = 0$

Differentiating w.r.t. x, we get

$$\Rightarrow 3x^2 - 4xy^2 - 4x^2y\frac{dy}{dx} + 5 + \frac{dy}{dx} = 0$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{3x^2 - 4xy^2 + 5}{4x^2y - 1}$$

$$y'(1) = \frac{3 - 4 + 5}{4 - 1} = \frac{4}{3}$$

Also, y''

$$= \frac{(6x - 4y^2 - 8xyy')(4x^2y - 1) - (8xy + 4x^2y')}{(4x^2y - 1)^2}$$

$\Rightarrow y''(1)$
$$= \frac{\left(6 - 4 - 8.\frac{4}{3}\right)(4 - 1) - \left(8 + 4.\frac{4}{3}\right)(3 - 4 + 5)}{(4 - 1)^2}$$

$$= -8\frac{22}{27}$$

101 **(b,c,d)**
Let $f(x) = ax^3 + bx^2 + bx + d$
Then, $f(1) = 0$ and $f'(1) = 0$
$\Rightarrow \ a + 2b + d = 0$ ... (i)
and $3a + 2b + b = 0$ ... (ii)
From Eqs. (i) and (ii) we get
$a = d = -b$

102 **(a,b,d)**

$$f(x)=\frac{\sqrt{\left(\sqrt{x-1}\right)^2+1-2\sqrt{x-1}}}{\sqrt{x-1}-1}x$$

$$=\frac{\left|\sqrt{x-1}-1\right|}{\sqrt{x-1}-1}x$$

$$=\begin{cases}-x \text{ if } x\in[1,2)\\ x \text{ if } x\in(2,\infty)\end{cases}$$

103 **(a,b,c)**

Given, $F(x)=f(x).g(x)$ ...(i)

On differentiating both sides w.r.t. x, we get

$F'(x)=f'(x).g(x)+g'(x).f(x)$

$\Rightarrow F'(x)=f(x)g'(x)\left[\frac{f(x)}{f'(x)}+\frac{g(x)}{g'(x)}\right]$

$\Rightarrow F'=c'\left[\frac{f}{f'}+\frac{g}{g'}\right]$

Therefore, option (a) is correct.

Again, on differentiating both sides w.r.t. x, we get

$F''(x)=f''(x)g(x)+g''(x).f(x)+2f'(x).g'(x)$

$\Rightarrow F''(x)=f''(x).g(x)+g''(x).f(x)+2c$ ... (ii)

On dividing both sides by $F(x)=f(x).g(x)$

$\{\because f'(x).g'(x)=c\}$

Then, $\frac{F''(x)}{F(x)}=\frac{F''(x)}{f(x)}+\frac{g''(x)}{g(x)}+\frac{2c}{f(x)g(x)}$

or $\frac{F''}{F}=\frac{f''}{f}+\frac{g''}{g}+\frac{2c}{fg}$

Therefore, option (b) is correct.

Also, given $f'(x)g'(x)=c$

On differentiating both sides w.r.t. x, we get

$f'(x)g''(x)+g'(x)f''(x)=0$

From Eq. (ii)

$F''=f''(x).g(x)+g''(x).f(x)+2c$

On differentiating both sides w.r.t. x, we get

$$F'''(x)=f''(x).g'(x)+f'''(x).g(x)+g''(x).f'(x)+f(x).g'''(x)+0$$

$=f'''(x).g(x)+g'''(x).f(x)+0$

Now, on dividing both sides by $F(x)=f(x)g(x)$

Then, $\frac{F'''(x)}{F(x)}=\frac{f'''(x)}{f(x)}+\frac{g'''(x)}{g(x)}$

or $\frac{F'''}{F}=\frac{f'''}{f}+\frac{g'''}{g}$

Therefore, option (c) is correct.

104 **(a,b,c)**

We have $\sin^{-1}(\cos x)=\frac{\pi}{2}-\cos^{-1}(\cos x)$

$$=\begin{cases}\frac{\pi}{2}-x, & \text{if } 0<x\le\pi\\ \frac{\pi}{2}-(2\pi-x), & \text{if } \pi<x<2\pi\end{cases}$$

$$=\begin{cases}\frac{\pi}{2}-x, & \text{if } 0<x\le\pi\\ x-\frac{3\pi}{2}, & \text{if } \pi<x<2\pi\end{cases}$$

$$\therefore \frac{d}{dx}\{\sin^{-1}(\cos x)\}=\begin{cases}-1, & \text{if } 0<x<\pi\\ 1, & \text{if } \pi<x<2\pi\end{cases}$$

We have $\cos^{-1}(\sin x)=\frac{\pi}{2}-\sin^{-1}(\sin x)$

$$=\begin{cases}\frac{\pi}{2}-x, & \text{if } -\frac{\pi}{2}<x\le\frac{\pi}{2}\\ \frac{\pi}{2}-(\pi-x) & \text{if } \frac{\pi}{2}<x<\frac{3\pi}{2}\end{cases}$$

$$=\begin{cases}\frac{\pi}{2}-x, & \text{if } -\frac{\pi}{2}<x\le\frac{\pi}{2}\\ x-\frac{\pi}{2}, & \text{if } \frac{\pi}{2}<x<\frac{3\pi}{2}\end{cases}$$

$$\therefore \frac{d}{dx}(\cos^{-1}(\sin x))=\begin{cases}-1 & \text{if } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ 1, & \text{if } \frac{\pi}{2}<x<\frac{3\pi}{2}\end{cases}$$

105 **(a,c)**

$f(x-y), f(x)f(y)$ and $f(x+y)$ are in A.P.

$\Rightarrow f(x+y)+f(x-y)=2f(x)f(y)$ for all x,y

Putting $x=0, y=0$ in (1)

We get $f(0)+f(0)=2f(0)f(0)$

$\Rightarrow f(0)=1$ $(\because f(0)\neq 0)$

Putting $x=0, y=x$,

We get $f(x)+f(-x)=2f(0)f(x)$

$\Rightarrow f(x)=f(-x)$ (1)

$\Rightarrow f(4)=f(-4), f(3)=f(-3)$

Differentiating (1) w.r.t. x, $f'(x)+f'(-x)=0$

$\Rightarrow f'(4)+f'(-4)=0$

106 **(b,d)**

$y=x^{(\log x)^{\log(\log x)}}$

$\Rightarrow \log y=(\log x)(\log x)^{\log(\log x)}$ (1)

Taking log of both sides, we get

$\Rightarrow \log(\log y)=\log(\log x)+\log(\log x)\log(\log x)$

Differentiating w.r.t. x, we get

$$\frac{1}{\log y}.\frac{1}{y}\frac{dy}{dx}=\frac{1}{x\log x}+\frac{2\log(\log x)}{\log x}\frac{1}{x}$$

$$=\frac{2\log(\log x)+1}{x\log x}$$

$$\Rightarrow\frac{dy}{dx}=\frac{y}{x}.\frac{\log y}{\log x}(2\log(\log x)+1)$$

Substituting the value of y from (1), we get

$$\frac{dy}{dx}=\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x)+1)$$

107 **(a,c,d)**

$$y^2=x+y\Rightarrow\frac{dy}{dx}=\frac{1}{2y-1}$$

Also $y=\frac{x}{y}+1\Rightarrow\frac{dy}{dx}=\frac{y}{2x+y}$

Also $y^2-y-x=0\Rightarrow y=\frac{1\pm\sqrt{1+4x}}{2}$

$$\Rightarrow y=\frac{1+\sqrt{1+4x}}{2}.\ \ (\text{as } y>0)$$

$$\Rightarrow y'=\frac{1}{4}\frac{4}{\sqrt{1+4x}}=\frac{1}{\sqrt{1+4x}}$$

108 **(b,c,d)**

$\frac{d}{dx}\int_{x^2}^{x^3}f(t)dt=f(x^3).3x^2-f(x^2).2x$
$=\text{In}\,x^3.3x^2-\text{In}\,x^2.2x$
$=9x^2\text{In}\,x-4x\,\text{In}\,x$
$=x\,\text{In}\,x(9x-4)$
Let $z=x\,\text{In}\,x(9x-4)$
Then, $\frac{dz}{dx}=(1+\text{In}\,x)(9x-4)+9x\,\text{In}\,x$
At $x=e,\frac{dz}{dx}=2(9e-4)+9e=27e-8$

109 **(a,c,d)**

$$f'(x)=\lim_{x\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{f\left(1+\frac{h}{x}\right)}{\frac{h}{x}x}=\frac{f'(1)}{x}=\frac{1}{x}$$

$\Rightarrow f(x)=\text{In}\,x$ as $f(1)=0$

110 **(a)**

Given $u=f(\tan x)$

$$\Rightarrow\frac{du}{dx}=f'(\tan x)\sec^2 x$$

and $v=\text{g}(\sec x)$

$$\Rightarrow\frac{dv}{dx}=\text{g}'(\sec x)\sec x\tan x$$

$$\therefore\frac{du}{dv}=\frac{(du/dx)}{(dv/dx)}=\frac{f'(\tan x)}{\text{g}'(\sec x)}.\frac{1}{\sin x}$$

$$\therefore\left(\frac{du}{dv}\right)_{x=\pi/4}=\frac{f'(1)}{\text{g}'(\sqrt{2})}.\sqrt{2}$$

$$=\frac{2}{4}.\sqrt{2}=\frac{1}{\sqrt{2}}$$

111 **(a)**

Since $|f(x)-f(y)|\le|x-y|^3$, where $x\neq y$

$$\therefore\left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|^2$$

Taking lim as $y\to x$, we get

$$\lim_{y\to x}\left|\frac{f(x)-f(y)}{x-y}\right|\le\lim_{y\to x}|x-y|^2$$

$$\Rightarrow\left|\lim_{y\to x}\frac{f(x)-f(y)}{x-y}\right|\le\left|\lim_{y\to x}(x-y)^2\right|$$

$$\Rightarrow|f'(x)|\le0$$

$$\Rightarrow|f'(x)|=0\ \ (\because|f'(x)|\ge0)$$

$$\therefore f'(x)=0$$

$$\Rightarrow f(x)=c\ (\text{constant})$$

112 **(a)**

$f(x)+f(x-2)=0$ (1)

Replace x by $x-2\Rightarrow f(x-2)+f(x-4)=0$
(2)

From (1) and (2), $f(x)-f(x-4)=0$

Replace x by $x+4\ \Rightarrow f(x+4)=f(x)$

$$\Rightarrow f(x)=f(x+4)=f(x+8)=\ldots=f(x+4000)$$

$$\Rightarrow f'(x)=f'(x+4000)$$

Hence, both the statements are true and

Statement 2 is correct explanation of Statement 1

Hence, $f(x)$ is periodic with period 4

113 **(c)**
Fro $0 < x < 1$,

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\therefore \frac{dv}{dv} = 1 \quad (\because u = v)$

114 **(b)**
Both the statement are true, but Statement 2 is not correct explanation of Statement 1

Statement 1 is true as period of $\sin x$ is 2π

Or, in general if for $y = f(x), f(a) = f(b)$, we cannot say $f'(a) = f'(b)$

115 **(d)**
For $x < 0, \frac{d}{dx}(\text{In}|x|) = \frac{d}{dx}(\text{In}(-x))$

$$= \frac{1}{(-x)}(-1) = \frac{1}{x}$$

116 **(d)**
Statement 2 is true as $f(\alpha) = 0$ and $f'(\alpha) = 0$, then definitely α is repeated root of $f(x) = 0$

But from data, we are not sure how many times a root repeats

Also $f(x) = (x-\alpha)^n \times g(x)$, which changes sign at $x = \alpha$, when n is odd and does not if n is even. Hence, Statement 1 is false

117 **(a)**
1. Given, $f(x) = ax^{41} + bx^{-40}$

$f'(x) = 41ax^{40} - 40bx^{-41}$

$f''(x) = 1640\, ax^{39} + 1640bx^{-42}$

Now, $\frac{f''(x)}{f(x)} = \frac{1640(ax^{39}+bx^{-42})}{ax^{41}+bx^{-40}} = 1640x^{-2}$

2. $\frac{d}{dx}\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{d}{dx}\tan^{-1}(\tan 2x)$

$$= \frac{d}{dx}\, 2x$$

$$= 2$$

$\therefore$ Statement I is true, but II is false.

118 **(a)**
$\because e^{xy} + \log(xy) + \cos(xy) + 5 = 0$

$$\therefore e^{xy}\frac{d}{dx}(xy) + \frac{1}{(xy)}\frac{d}{dx}(xy) - \sin(xy)\frac{d}{dx}(xy) = 0$$

$$\Rightarrow \frac{d}{dx}(xy)\left\{e^{xy} + \frac{1}{xy} - \sin(xy)\right\} = 0$$

$$\because e^{xy} + \frac{1}{xy} - \sin(xy) \neq 0$$

$$\therefore \frac{d}{dx}(xy) = 0$$

$$\Rightarrow x\frac{dy}{dx} + y.1 = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

119 **(a)**

$$f(x) = x[x] = \begin{cases} -x, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ x, & 1 \le x < 2 \\ 2x, & 2 \le x < 3 \\ \dots & \dots \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -1, & -1 < x < 0 \\ 0, & 0 < x < 1 \\ 1, & 1 < x < 2 \\ 2, & 2 < x < 3 \\ \dots & \dots \end{cases} \Rightarrow f'(x) = [x]$$

120 **(c)**
Statement 1 is always true, but Statement 2 is not always true, as if $f'(x) = \cos x$, then $f(x)$ can be $\sin x$ which is odd function, but if $f(x) = -\sin x + 2$, then $f(x)$ is neither odd nor even

121 **(a)**
Given $f(x+y^3) = f(x) + f(y^3) \forall\, x, y \in R$

Put $x = y = 0$, we get $f(0+0) = f(0) + f(0) \Rightarrow f(0) = 0$

Now, put $y = -x^{1/3}$, we get $f(0) = f(x) + f(-x)$

$\Rightarrow f(x) + f(-x) = 0$

$\Rightarrow f(x)$ is an odd function

$\Rightarrow f'(x)$ is an even function

$\Rightarrow f(-2) = a$

122 **(b)**

a. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12t^2-6t-18}{5t^4-15t^2-20}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = \frac{12-6-18}{5-15-20} = \frac{2}{5}$

$\Rightarrow 5\left.\frac{dy}{dx}\right|_{t=1} = 2$ at $t = 1$

b. Let us take $P(x) = a(x-2)^4 + b(x-2)^3 + c(x-2)^2 + d(x-2) + e$

$-1 = P(2) = e$

$0 = P'(2) = d$

$2 = P''(2) = 2c \Rightarrow c = 1$

$-12 = P'''(2) = 6b \Rightarrow b = -2$

$24 = p'''(2) = 24a \Rightarrow a = 1$

Thus, $P''(x) = 12(x-2)^2 - 12(x-2) + 2$

$\Rightarrow P''(3) = 12 - 12(1) + 2 = 2$

c. Here $\sqrt{(1+y^4)} = \sqrt{\left(1+\frac{1}{x^4}\right)} = \frac{\sqrt{1+x^4}}{x^2}\ \left(\because y = \frac{1}{x}\right)$

$\Rightarrow \frac{\sqrt{1+y^4}}{\sqrt{1+x^4}} = \frac{1}{x^2}$ (1)

But $y = \frac{1}{x}$

$\therefore \frac{dy}{dx} = -\frac{1}{x^2}$ (2)

From (1) and (2), $\frac{\sqrt{1+y^4}}{\sqrt{1+x^4}} = -\frac{dy}{dx}$

$\Rightarrow \frac{\frac{dy}{\sqrt{1+y^4}}}{\frac{dx}{\sqrt{1+x^2}}} = -1$

d. obviously, $f(x)$ is a linear function

Also from $f'(0) = p$ and $f(0) = q, f(x) = px + q$

$\Rightarrow f''(0) = 0$

123 **(a)**

a. $f(1-x) = f(1+x)$

$\Rightarrow -f'(1-x) = f'(1+x)$

Hence, graph of $f(x)$ is symmetrical about point (1, 0) (as if $f(x) = -f(-x)$, then $f(x)$ is odd its graph is symmetrical about (0, 0). Now shift the graph at (1,0))

b. $f(2-x) + f(x) = 0$

Replace x by $1+x$,then $f\big(2-(1-x)\big) + f(1+x) = 0$

$\Rightarrow f(1-x) + f(1+x) = 0$

$\Rightarrow -f'(1-x) + f'(1+x) = 0$

$\Rightarrow f'(1-x) = f'(1+x)$ (1)

$\Rightarrow$ Graph of $f'(x)$ is symmetrical about line $x = 1$

Also, put $x = 2$ in (1), we get $f'(-1) = f'(3)$

c. $f(x+2) + f(x) = 0$ (1)

Replace x by $x+2$, we get $f(x+4) + f(x+2) = 0$ (2)

From (1) and (2), we have $f(x) = f(x+4)$

Hence $f(x)$ is periodic with period 4

Also, $f'(x) = f'(x+4)$. Hence $f'(x)$ is periodic with period 4

Put $x = -1$ in $f'(x) = f'(x+4)$, we get $f'(-1) = f'(3)$

d. Putting $x = 0, y = 0$, we get $2f(0) + \{f(0)\}^2 = 1$

$\Rightarrow f(0) = \sqrt{2} - 1\ \{\because f(0) > 0\}$

Putting $y = x, 2f(x) + \{f(x)\}^2 = 1$

Diff. w.r.t. x. we get

$2f,(x) + 2f(x).f'(x) = 0$ or $f'(x)\{1 + f(x)\} = 0$

$\Rightarrow f'(x) = 0$, because $f(x) > 0$

124 **(c)**

a. We know that

$2\tan^{-1} x$

$$= \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x > 1 \\ -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x < -1 \end{cases}$$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$ if $x < -1$ or $x > 1$

b. $\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \begin{cases} \tan^{-1} x, & x \ge 0 \\ -\tan^{-1} x, & x < 0 \end{cases}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{1+x^2}$ if $x < 0$

c. $y\left|e^{|x|} - e\right| = \begin{cases} |e^x - e|, & x \ge 0 \\ |e^{-x} - e|, & x < 0 \end{cases} =$

$$\begin{cases} e^x - e, & x \ge 1 \\ e - e^x, & 0 \le x < 1 \\ e - e^{-x}, & -1 \le x < 0 \\ e^{-x} - e, & x < -1 \end{cases}$$

$\Rightarrow \frac{dy}{dx} >$ if $x > 1$ of $-1 < x < 0$

d. $u = \log|2x|, v = |\tan^{-1} x|$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}, \text{and} \frac{dv}{dx} = \begin{cases} \frac{1}{1+x^2}, & x > 0 \\ -\frac{1}{1+x^2}, & x < 0 \end{cases}$$

$$\Rightarrow \frac{du}{dv} = \begin{cases} \frac{1+x^2}{x}, & x > 0 \\ -\frac{1+x^2}{x}, & x < 0 \end{cases}$$

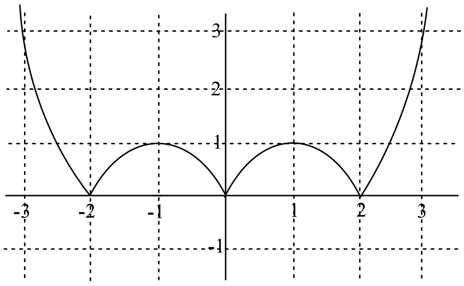
Now we know that $\frac{1+x^2}{x} = x + \frac{1}{x} > 2$ if $x > 1$ and < -2 if $x < -1$

$\Rightarrow \frac{du}{dv} > 2$ if $x < -1$ or $x > 1$

125 **(a)**

a. p,q,r

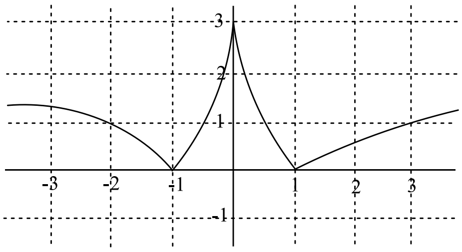
The graph of $y = |x^2 - 2|x||$



From the graph dy/dx is negative for p, q, r

b. q,s

The graph of $y = |\log |x||$



from the graph dy/dx is negative for q, s

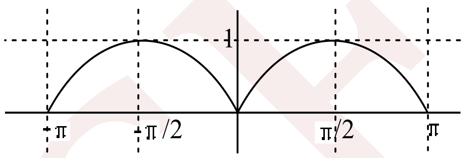
c. q,r

$$y = x[x/2] = \begin{cases} -x, & -4 \le x < -2 \\ -x, & -2 \le x < 0 \\ 0, & 0 \le x < 2 \\ x, & 2 \le x < 4 \end{cases}$$

Hence dy/dx is negative for q, r

d. q

The graph of $y = |\sin x|$



From the graph dy/dx is negative for q

126 **(b)**

$D^*(u.v) = D^*(f(x).\mathrm{g}(x))$

$$= \lim_{h\to 0} \frac{f^2(x+h)\mathrm{g}^2(x+h) - f^2(x)\mathrm{g}^2(x)}{h}$$

$$= \lim_{h\to 0} f^2(x+h)\left\{\frac{\mathrm{g}^2(x+h) - \mathrm{g}^2(x)}{h}\right\}$$

$$+\mathrm{g}^2(x)\frac{\{f^2(x+h) - f^2(x)\}}{h}$$

$= f^2(x)D^*\mathrm{g}(x) + \mathrm{g}^2(x)D^*f(x)$

$= u^2D^*v + v^2D^*u$

127 **(c)**

$\because y = e^{3x+7}$

$\therefore y_1 = 3e^{3x+7}, y_2 = 3^2e^{3x+7} \ldots$

$\therefore y_n(x) = 3^n.e^{3x+7}$,

Then, $y_n(0) = 3^n.e^7$

128 **(b)**

Suppose degree of $f(x) = n$, then degree of $f' = n-1$ and deg $f'' = n-2$,

So $n = n - 1 + n - 2$

Hence, $n = 3$

So put $f(x) = ax^3 + bx^2 + cx + d$. (where $a \ne 0$)

From $f(2x) = f'(x).f''(x)$,

We have $8ax^3 + 4bx^2 + 2cx + d$

$= (3ax^2 + 2bx + c)(6ax + 2b)$

$= 18\,a^2x^3 + 18abx^2 + (6ac + 4b^2)x + 2bc$

Comparing coefficients of terms, we have

$18a^2 = 8a \Rightarrow a = 4/9$

$18ab = 4b \Rightarrow b = 0$

$2c = 6ac + 4b^2 \Rightarrow c = 0$

$d = 2bc \Rightarrow d = 0$

$\Rightarrow f(x) = \frac{4x^3}{9}$, which is clearly one-one and onto

$\Rightarrow f(3) = 12$

Also, $\frac{4x^3}{9} = x \Rightarrow x = 0, x = \pm 3/2$

Hence sum of roots of equation is zero

129 **(d)**

Here, $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

Put $f'(1) = a, f''(2) = b, f'''(3) = c$ (1)

$\therefore f(x) = x^3 + ax^2 + bx + c$

$\Rightarrow f'(x) = 3x^2 + 2ax + b$, or

$f'(1) = 3 + 2a + b$ (2)

$\Rightarrow f''(x) = 6x + 2a$, or

$f''(2) = 12 + 2a$ (3)

$\Rightarrow f'''(x) = 6$, or

$f'''(3) = 6$ (4)

From(1)and(4), $c = 6$

From(1),(2) and (3), we have $a = -5, b = 2$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f'(x) = 3x^2 - 10x + 2$

130 **(b)**

From the given information, we have $f(x) = (x-c)^m\mathrm{g}(x)$, where $\mathrm{g}(x)$ is polynomial of degree $n - m$,

Then $x = c$ is common root for the equations $f(x) = 0, f^1(x) = 0, f^2(x) = 0, \ldots, f^{m-1}(x) = 0$,

Where $f^r(x)$ represent rth derivative of $f(x)$ w.r.t. x.

131 **(b)**

Since $1, a_1, a_2, \ldots, a_{n-1}$ are roots of $x^n - 1 = 0$, then

$x^n - 1 = (x-1)(x-a_1)(x-a_2)\ldots(x-a_{n-1})$ (1)

$$\Rightarrow \frac{x^n - 1}{x - 1} = (x-a_1)(x-a_2)\ldots(x-a_{n-1})$$

$$\Rightarrow \lim_{x\to 1}\frac{x^n - 1}{x-1} = \lim_{x\to 1}[(x-a_1)(x-a_2)\ldots(x - a_{n-1})]$$

$\Rightarrow (1-a_1)(1-a_2)\ldots(1-a_{n-1}) = n$

132 **(b)**

Here put $g'(1) = a, g''(2) = b$ (1)
Then $f(x) = x^2 + ax + b, f(1) = 1 + a + b \Rightarrow$
$f'(x) = 2x + a, f''(x) = 2$
$\therefore g(x) = (1 + a + b)x^2 + (2x + a)x + 2$
$= x^2(3 + a + b) + ax + 2$
$\Rightarrow g'(x) = 2x(3 + a + b) + a$ and $g''(x) = 2(3 + a + b)$
Hence, $g'(1) = 2(3 + a + b) + a$ (2)
$g''(2) = 2(3 + a + b)$ (3)
From (1), (2) and (3), we have $a = 2(3 + a + b) + a$
and $b = 2(3 + a + b)$
$\Rightarrow 3 + a + b = 0$ and $b + 2a + 6 = 0$
Hence, $b = 0$ and $a = -3$. So, $f(x) = x^2 - 3x$ and $g(x) = -3x + 2$
$\sqrt{\frac{f(x)}{g(x)}} = \sqrt{\frac{x^2-3x}{-3x+2}}$ is defined if $\frac{x^2-3x}{-3x+2} \geq 0$
$\Rightarrow \dfrac{x(x-3)}{(x-2/3)} \leq 0 \Rightarrow x \epsilon (-\infty, 0] \cup (2/3, 3]$

133 **(d)**

$g(x + y) = g(x) + g(y) + 3x^2y + 3xy^2$ (1)
$\Rightarrow g'(x + y) = g'(x) + 6yx + 3y^2$ (differentiating w.r.t. x keeping y as constant)
Put $x = 0$
$\Rightarrow g'(y) = g'(0) + 3y^2$
$\Rightarrow g'(y) = -4 + 3y^2$
$\Rightarrow g'(x) = -4 + 3x^2$
$\Rightarrow g(x) = -4x + x^3 + c$
Now put $x = y = 0$ in (1), we get $g(0) + g(0) + 0$
$\Rightarrow g(0) = 0$
$\Rightarrow g(x) = x^3 - 4x$
$g(x) = 0 \Rightarrow x^3 - 4x = 0 \Rightarrow x = 0, 2, -2$. Hence, three roots
$\sqrt{g(x)} = \sqrt{x^3 - 4x}$ is defined if $x^3 - 4x \geq 0$ or $x \in [-2, 0] \cup [2, \infty)$
Also, $g'(x) = 3x^2 - 4 \Rightarrow g'(1) = -1$

134 **(d)**

$x = f(t) = a^{\text{In}(b^t)} = a^{t \text{ In} b}$ (1)
$y = g(t) = b^{-\text{In}(a^t)} = \left(b^{\text{In } a}\right)^{-t} = \left(a^{\text{In } b}\right)^{-y}$
$= a^{-t \text{ In } b}$
$\therefore y = g(t) = a^{\text{In}(b^{-t})} = f(-t)$
From equations (1) and (2)
$xy = 1$
$\because y = \dfrac{1}{x}$
$\therefore \dfrac{dy}{dx} = -\dfrac{1}{x^2} = -\dfrac{1}{f^2(t)}$
Also, $xy = 1 \Rightarrow -\frac{1}{f^2(t)} = -g^2(t)$
$\therefore \dfrac{dy}{dx} = -\dfrac{1}{x^2} = -\dfrac{y^2}{1}$
Also, $xy = 1 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = -\frac{g(t)}{f(t)}$

135 **(2)**

Limit is $f'(e)$ where $f(x) = x^{\text{In}x} = e^{\text{In}^2 x}$
$\Rightarrow g'\big(f(x)\big)f'(x)e^{\text{In}^2 x} \cdot \dfrac{2 \text{ In } x}{x}$
$\Rightarrow f'(e) = e \cdot \dfrac{2}{e} = 2$

136 **(5)**

According to question $(a^2 - 2a - 15)e^{ax} + (b^2 - 2b - 15)e^{bx} = 0$
$\Rightarrow (a^2 - 2a - 15) = 0$ and $b^2 - 2b - 15 = 0$
$\Rightarrow (a - 5)(a + 3) = 0$ and $(b - 5)(b + 3) = 0$
$\Rightarrow a = 5$ or -3 and $b = 5$ or -3
$\therefore a \neq b$ hence $a = 5$ and $b = -3$
Or $a = -3$ and $b = 5$
$\Rightarrow ab = -15$

137 **(3)**

$y = \dfrac{x^4 - (x^2 + 2x + 1)}{x^2 - x - 1} = x^2 + x + 1$
$\therefore \dfrac{dy}{dx} = 2x + 1 = ax + b$
Hence $a = 2$ and $b = 1$

138 **(5)**

$y = \dfrac{a + bx^{3/2}}{x^{5/4}}$
$\Rightarrow y' \dfrac{\frac{3}{2}bx^{1/2}x^{5/4} - \frac{5}{4}x^{1/4}(a + bx^{3/2})}{x^{5/2}}$
According to the question,
$0 = \dfrac{\frac{3}{2}b5^{1/2}5^{5/4} - \frac{5}{4}5^{1/4}(a + b5^{3/2})}{5^{5/2}}$
$\Rightarrow \dfrac{3b}{2}5^{7/4} - a\dfrac{5^{5/4}}{4} - 5b\dfrac{5^{7/4}}{4} = 0$
$\Rightarrow b5^{7/4} = a5^{5/4}$
$\Rightarrow b\sqrt{5} = a$
$\Rightarrow a : b = \sqrt{5} : 1$

139 **(7)**

$g'(0) = b = \lim_{x \to 0} \dfrac{x^2 + x \tan x - x \tan 2x}{x(ax + \tan x - \tan 3x)}$
$= \lim_{x \to 0} \dfrac{x + \tan x - \tan 2x}{ax + \tan x - \tan 3x}$
$x + \left(x + \dfrac{x^3}{3} + \dfrac{2}{15}x^5 + \cdots \infty\right)$
$= \lim_{x \to 0} \dfrac{-\left(2x + \frac{8x^3}{3} + \frac{2}{15} \cdot 32x^5 + \cdots\right.}{ax + \left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots \infty\right) - \left(3x + \frac{27x^3}{3} + \right.}$
$= \lim_{h \to 0} \dfrac{x^3\left(-\frac{7}{3} + \frac{-62}{15}x^2 + \cdots \infty\right)}{(a + 1 - 3)x + \left(\frac{1}{3} - 9\right)x^3 + \frac{2}{15}(-242)x^5 +}$
b can be finite if $a + 1 - 3 = 0$
$\therefore a = 2$ and $b = \frac{-\frac{7}{3}}{\frac{1}{3} - 9} = \left(\frac{-7}{3}\right)\left(\frac{3}{-26}\right) = \frac{7}{26} \Rightarrow 52\frac{b}{a} = 7$

140 **(2)**

Since $f(x)$ is odd. Therefore $f(-x) = -f(x) \Rightarrow$
$f'(-x)(-1) = -f'(x)$
$\Rightarrow f'(-x) = f'(x) \therefore f'(-3) = f'(3) = -2$

141 **(9)**

$\frac{d}{dx}\{[(f(x)]^2 - [\phi(x)]^2\}$
$= 2[f(x).f'(x) - \phi(x).\phi'(x)]$
$= 2[f(x).\phi(x) - \phi(x).f(x)][\because f'(x) = \phi(x)$ and $\phi'(x) = f(x)]$
$= 0$
$\Rightarrow [f(x)]^2 - [\phi(x)]^2 =$ constant
$\therefore [f(10)]^2 - [\phi(10)]^2$
$= [f(3)]^2 - [\phi(3)]^2 - [f(3)]^2 - [f'(3)]^2 = 25 - 16 = 9$

142 **(9)**

$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x+0)}{h}$
$\Rightarrow \lim_{h\to 0}\frac{2f(x)+xf(h)+h\sqrt{f(x)}-2f(x)-xf(0)-0\sqrt{f(x)}}{h}$ as $f(0) = 0$
$\Rightarrow \lim_{h\to 0}\left(\frac{f(h) - f(0)}{h-0}\right) + \sqrt{f(x)} = f'(0) + \sqrt{f(x)}$
$\Rightarrow f'(x) = \sqrt{f(x)}\ (\because f'(0) = 0)$
$\Rightarrow \int \frac{f'(x)}{\sqrt{f(x)}}dx = \int dx$
$\Rightarrow 2\sqrt{f(x)} = x + c$
$\Rightarrow f(x) = \frac{x^2}{4}\ (\because f(0) = 0)$

143 **(6)**

$g(x) = f\left(-x + f(f(x))\right);\ f(0) = 0;\ f'(0) = 2$
$g'(x) = f'\left(-x + f(f(x))\right) \cdot [-1 + (f'(x)) \cdot f'(x)]$
$g'(0) = f'(f(0)) \cdot [-1 + f'(0) \cdot f'(0)]$
$= f'(0)[-1 + (2)(2)]$
$= (2)(3) = 6$

144 **(5)**

$z = (\cos x)^5; y = \sin x$
$\frac{dz}{dx} = -5\cos^4 x \cdot \sin x;\ \frac{dy}{dx} = \cos x$
$\therefore \frac{dz}{dy} = -5\cos^3 x \cdot \sin x$
Now $\frac{d^2z}{dy^2} = \frac{d}{dx}\left(\frac{dz}{dy}\right) \cdot \frac{dx}{dy}$
$= -5\frac{d}{dx}[\cos^3 x \cdot \sin x]\frac{1}{\cos x}$
$= -5(\cos^3 x - 3\sin^2 x \cdot \cos x)$
$= -5(\cos^3 x - 3\cos x\,(1 - \cos^2 x))$
$= -5(4\cos^3 x - 3\cos x)$
$= -5\cos 3x$
$\therefore \left.\frac{d^2z}{dy^2}\right|_{x=\frac{2\pi}{9}} = -5\cos 120° = \frac{5}{2}$

145 **(9)**

Let degree of $f(x)$ is n; degree of $f'(x) = n - 1$
degree of $f''(x)$ is $(n - 2)$
Hence $n = (n-1) + (n-2) = 2n - 3$
$\therefore n = 3$
Hence $f(x) = ax^3 + bx^2 + cx + d\ (a \neq 0)$
$f'(x) = 3ax^2 + 2bx + c$
$f''(x) = 6ax + ab$
$\therefore ax^3 + bx^2 + cx + d$
$= (3ax^2 + 2bx + c)(6ax + ab)$
$\therefore 18a^2 = a \Rightarrow a = \frac{1}{18}$

146 **(3)**

$f(x) \times f'(-x) = f(-x) \times f'(x)$
$\Rightarrow f'(x) \times f(-x) - f(x) \times f'(-x) = 0$
$\Rightarrow \frac{d}{dx}[f(x)f(-x)] = 0$
$\Rightarrow f(x)f(-x) = k$
Given $\left(f(0)\right)^2 = k = 9 \Rightarrow k = 9$
Then $f(3)f(-3) = 9 \Rightarrow f(-3) = 3$

147 **(1)**

$\frac{dx}{dt} = -\frac{3}{t^4} - \frac{2}{t^3} = -\left(\frac{3+2t}{t^4}\right)$
$\frac{dx}{dt} = -\left(\frac{3}{t^3} + \frac{2}{t^2}\right) = -\left(\frac{3+2t}{t^3}\right)$
$\Rightarrow \frac{dy}{dx} = t$
$\Rightarrow \frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3 = t - \left(\frac{1+t}{t^3}\right) \cdot t^3 = -1$

148 **(2)**

We have $g(x) = f(x)\sin x$ (1)
On differentiating equation (2) w.r.t. x., we get
$g'(x) = f(x)\cos x + f'(x)\sin x$ (2)
Again differentiating equation (2) w.r.t. x., we get
$g''(x) = f(x)(-\sin x) + f'(x)\cos x + f'(x)\cos x + f''(x)\sin x$ (3)
$\Rightarrow g''(-\pi) = 2f'(-\pi)\cos(-\pi) = 2 \times 1 \times (-1)$
$= -2$
Hence $g''(-\pi) = -2$

149 **(3)**

We have $f(5 - x) = -f(5 + x) \Rightarrow -f'(5 - x) = -f'(5 + x)$
$\Rightarrow f'(5 - 2) = f'(5 + 2) \Rightarrow f'(3) = f'(7) = 3$

150 **(8)**

$In(f(x)) = In(x-1) + In(x-2) + \cdots + In(x-n)$

$\Rightarrow f'(x) = f(x)\left[\frac{1}{x-1} + \frac{1}{x-2} + \cdots + \frac{1}{x-n}\right]$

$\Rightarrow f'(x) = (x-2)(x-3)(x-n) + (x-1)(x-3) \ldots (x-n) + \cdots + (x-1)\,(x-2) \ldots (x-(n-1))$

$\Rightarrow f'(n) = (n-1)(n-2)(n-3) \cdot 3.2.1$ (all other factors except the last vanishes when $x = n$)

$\Rightarrow 5040 = (n-1)!$

$\Rightarrow n = 8$

151 **(5)**

We have $(gof)(x) = x$

$\Rightarrow g'(f(x))f'(x) = 1$

When $f(x) = -\frac{7}{6} \Rightarrow x = 1$

$\Rightarrow g'(f(x))g'\left(-\frac{7}{6}\right)f'(1) = 1$

Hence $g'\left(-\frac{7}{6}\right) = \frac{1}{f'(1)} = \frac{1}{5}$

152 **(5)**

Here $x = \alpha$ is a repeated root of the equation $f(x) = 0$ hence $x = \alpha$ is also a root of the equation $f'(x) = 0$ i.e., $3x^2 + 6x - 9 = 0$

Or $x^2 + 2x - 3 = 0$ or $(x+3)(x-1) = 0$

has the root α once which can be either -3, or 1

If $\alpha = 1$, then $f(x) = 0$ gives $c - 5 = 0$ or $c = 5$

If $\alpha = -3$, then $f(x) = 0$ gives $-27 + 27 + 27 + c = 0 \therefore c = -27$