LINEAR INEQUALITIES

Single Correct Answer Type

1. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation
   a) $0 \le M \le 1$ b) $1 \le M \le 2$ c) $2 \le M \le 3$ d) $3 \le M \le 4$
2. The product of n positive numbers is unity. Then their sum is
   a) A positive integer b) Divisible by n c) Equals to $n + 1/n$ d) Never less than n
3. If $a, b, c, d \in R^+ - \{1\}$, then the minimum value of $\log_d a + \log_b d + \log_a c + \log_c b$ is
   a) 4 b) 2 c) 1 d) None of these
4. For $x^2 - (a + 3)|x| + 4 = 0$ to have real solutions, the range of a is
   a) $(-\infty, -7] \cup [1, \infty)$ b) $(-3, \infty)$ c) $(-\infty, -7)$ d) $(1, \infty)$
5. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $\min f(x) > \max g(x)$, then the relation between b and c, is
   a) No real value of b and c b) $0 < c < b\sqrt{2}$ c) $|c| < |b|\sqrt{2}$ d) $|c| > |b|\sqrt{2}$
6. If $a, b, c, \in R^2$, then $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is always
   a) ≥ 12 b) ≥ 9 c) ≤ 12 d) None of these
7. If $a, b, c, \in R^+$ such that $a + b + c = 18$, then the maximum value of $a^2b^3c^4$ is equal to
   a) $2^{18} \times 3^2$ b) $2^{18} \times 3^3$ c) $2^{19} \times 3^2$ d) $2^{19} \times 3^3$
8. The minimum value of $\frac{x^4+y^4+z^2}{xyz}$ for positive real numbers x, y, z is
   a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $4\sqrt{2}$ d) $8\sqrt{2}$
9. If $y = 3^{x-1} + 3^{-x-1}$, then the least value of y is
   a) 2 b) 6 c) 2/3 d) 3/2
10. The least value of the expression $2\log_{10} x - \log_x(0 \cdot 01)$, for $x > 1$, is
    a) 10 b) 2 c) $-0 \cdot 01$ d) None of these
11. If $a > 0$, then least value of $(a^3 + a^2 + a + 1)^2$ is
    a) $64a^2$ b) $16a^4$ c) $16a^3$ d) None of these
12. If l, m, n be the three positive roots of the equation $x^3 - ax^2 + bx - 48 = 0$, then the minimum value of $(1/l) + (2/m) + (3/n)$ equals
    a) 1 b) 2 c) 3/2 d) 5/2
13. If $a, b, c, \in R^+$, then the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is equal to
    a) abc b) $2abc$ c) $3abc$ d) $6abc$
14. If $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in A. P. $(a, b, c > 0)$, then the minimum value of $a + b + c$ is
    a) 1 b) 3 c) 5 d) 9
15. Minimum value of $(b + c) / a + (c + b) / b + (a + b) / c$ (for real positive numbers a, b, c) is
    a) 1 b) 2 c) 4 d) 6
16. If a, b, c, R^2, then $\frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b}$ is always
    a) $\le \frac{1}{2}(a + b + c)$ b) $\ge \frac{1}{3}\sqrt{abc}$ c) $\le \frac{1}{3}(a + b + c)$ d) $\ge \frac{1}{3}\sqrt{abc}$
17. If a, b, c are positive real numbers such that $b + c - a, c + a - b$ and $a + b - c$ are positive, then $(b + c - a)(c + a - b)(a + b - c) - abc$ is
    a) Positive b) Negative c) Non-positive d) Non-negative
18. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is
    a) $3abc$ b) $6abc$ c) abc d) $4abc$
19. If $a, b, c, d \in R^+$ and a, b, c, d are in H.P., then
    a) $a + d > b + c$ b) $a + b > c + d$ c) $a + c > b + d$ d) None of these

20. If the product of n positive is n^n, then their sum is
    a) a positive integer b) Divisible by n c) Equal to $n+1/n$ d) Never less than n^2
21. $f(x)=\frac{(x-2)(x-1)}{(x-3)}, \forall\, x>3$. The minimum value of $f(x)$ is equal to
    a) $3+2\sqrt{2}$ b) $3+2\sqrt{3}$ c) $3\sqrt{2}+2$ d) $3\sqrt{2}-2$
22. A rod fixed length k slides along the coordinate axes. If it meets the axes at $A(a,0)$ and $B(0,b)$, then the minimum value of $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2$ is
    a) 0 b) 8 c) $k^2-4+\dfrac{4}{k^2}$ d) $k^2+4+\dfrac{4}{k^2}$
23. If positive numbers a,b,c be in H.P., then equation $x^2-kx+2b^{101}-a^{101}-c^{101}=0\ (k\in R)$ has
    a) Both roots positive b) Both roots negative
    c) One positive and one negative root d) Both roots imaginary
24. If a,b,c are the sides of a triangle, then the minimum value of $\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}$ is equal to
    a) 3 b) 6 c) 9 d) 12
25. The least value of $6\tan^2\phi+54\cot^2\phi+18$ is
    (I) 54 when A. M. $\geq$ G. M. is applicable for $6\tan^2\phi, 54\cot^2\phi, 18$
    (II) 54 whenA M. $\geq$ G. M. is applicable for $6\tan^2\phi, 54\cot^2\phi$ and 18 is added future (III) 78 when $\tan^2\phi=\cot^2\phi$
    a) (I) is correct, II is false b) (I) and (II) are correct
    c) (III) is correct d) None of the above are correct

Multiple Correct Answers Type

26. If A is area and $2s$ the sum of the sides of triangle, then
    a) $A\leq\dfrac{s^2}{4}$ b) $A<\dfrac{s^2}{3\sqrt{3}}$ c) $A<\dfrac{s^2}{\sqrt{3}}$ d) None of these
27. For positive real numbers a,b,c such that $a+b+c=p$, which one holds?
    a) $(p-a)(p-b)(p-c)\leq\dfrac{8}{27}p^3$ b) $(p-a)(p-b)(p-c)\geq 8abc$
    c) $\dfrac{bc}{a}+\dfrac{ca}{b}+\dfrac{ab}{c}\leq p$ d) None of these
28. Let a_1,a_2,a_3 be any position real numbers, then which of the following statement is true?
    a) $3a_1a_2a_3\leq a_1^3+a_2^3+a_3^3$ b) $\dfrac{a_1}{a_2}+\dfrac{a_2}{a_3}+\dfrac{a_3}{a_1}\geq 3$
    c) $(a_1+a_2+a_3)\left(\dfrac{1}{a_1}+\dfrac{1}{a_2}+\dfrac{1}{a_3}\right)\geq 9$ d) $(a_1+a_2+a_3)\left(\dfrac{1}{a_1}+\dfrac{1}{a_2}+\dfrac{1}{a_3}\right)^3\leq 27$
29. If first and $(2n-1)^{\text{th}}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are a,b,c, respectively, then
    a) $a=b=c$ b) $a+c=b$ c) $a>b>c$ d) $ac-b^2=0$
30. If x,y,z are positive numbers in A.P., then
    a) $y^2\geq xz$ b) $xy+yz\geq 2xz$ c) $\dfrac{x+y}{2y-x}+\dfrac{y+z}{2y-z}\geq 4$ d) None of these
31. Which of the following is true?
    a) $2(a^3+b^3+c^3)\geq bc(b+c)+ca(c+a)+ab(a+b)$ b) $\dfrac{a^3+b^3+c^3}{3}>\dfrac{(a+b+c)(a^2+b^2+c^2)}{9}$
    c) $\dfrac{bc}{b+c}+\dfrac{ca}{c+a}+\dfrac{ab}{a+b}<\dfrac{1}{2}(a+b+c)$ d) $\dfrac{2}{b+c}+\dfrac{2}{c+a}+\dfrac{2}{a+b}<\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$

Linked Comprehension Type

This section contain(s) 3 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 32 to -32

If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$ are positive, then

32. Which has the greatest absolute value?
    a) b b) c c) d d) e

Paragraph for Question Nos. 33 to - 33

Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a, b, c are non-negative)

33. Minimum non-negative real value of a is
    a) 10 b) 9 c) 6 d) 4

Integer Answer Type

34. If x, y and z are positive real numbers and $x = \frac{12-yz}{y+z}$. The maximum value(xyz) equals
35. Let $x^2 - 3x + p = 0$ has two positive roots 'a' and 'b' then minimum value of $\left(\frac{4}{a} + \frac{1}{b}\right)$ is
36. For any $x, y \in R, xy > 0$ then the maximum value of $\frac{2x}{y^3} + \frac{x^3y}{3} + \frac{4y^2}{9x^4}$ is
37. For $x \geq 0$, the smallest value of the function $f(x) = \frac{4x^2+8x+13}{6(1+x)}$, is
38. If a, b and c are positive and $9a + 3b + c = 90$ then the maximum value of $(\log a + \log b + \log c)$ is (base of the logarithm is 10)
39. Given that x, y, z are positive real such that $xyz = 32$. If the minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is equal m then the value of $m/16$ is
40. If $x, y \in R^+$ satisfying $x + y = 3$, then the maximum value of x^2y is

: ANSWER KEY :

1)	a	2)	d	3)	a	4)	d
5)	d	6)	b	7)	d	8)	b
9)	c	10)	b	11)	c	12)	c
13)	d	14)	b	15)	d	16)	a
17)	b	18)	a	19)	a	20)	d
21)	a	22)	d	23)	c	24)	a
25)	b	1)	a,b.	2)	a, b	3)	
	a,b,c	4)	c, d.				
5)	a, c	6)	a,b,c,d	1)	c	2)	d
	1)	8	2)	3	3)	2	
	4)	2					
5)	3	6)	6	7)	4		

: HINTS AND SOLUTIONS :

1 **(a)**

As A.M. ≥ G.M. for positive real numbers, we get

$$\frac{(a+b)(c+d)}{2} \geq \sqrt{(a+b)(c+d)}$$

$\Rightarrow M \leq 1$

Also,

$(a+b)(c+d) > 0$ $[\because a, b, c, d > 0]$

$\therefore\ 0 \leq M \leq 1$

2 **(d)**

Let $x_1, x_2 \dots, x_n$ be the n positive numbers. Given that

$x_1 x_2 x_3 \dots\ x_n = 1$

We know for positive numbers,

A.M. ≥ G.M.

$$\Rightarrow \frac{x_1 + x_2 + \ldots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \dots x_n}$$

$$\Rightarrow \frac{x_1 + x_2 + \ldots + x_n}{n} \geq 1$$

$\Rightarrow\ x_1 + x_2 + \ldots + x_n \geq n$

3 **(a)**

A.M. ≥ G.M.

$$\Rightarrow \frac{\log_d a + \log_c b + \log_a c + \log_b d}{4}$$

$$\geq \sqrt[4]{\frac{\log a}{\log d} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} \times \frac{\log d}{\log b}}$$

$\Rightarrow \log_d a + \log_b d + \log_a c + \log_c b\ \geq 4$

4 **(d)**

$$a = \frac{x^2+4}{1x1} - 3$$

$$= |x| + \frac{4}{|x|} - 3$$

$$\geq 2\sqrt{|x| \times \frac{4}{|x|}} - 3\ (\because \text{A. M.} \geq \text{G. M.})$$

$\Rightarrow a \geq 1$

5 **(d)**

If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$

Then, $f(x)$ is minimum and $g(x)$ is maximum at

$$f(x) = \frac{-D}{4a},\ \left(\because x = -\frac{-b}{a} \text{ and } f(x) = \frac{-D}{4a}\right)$$

$$\therefore\ \min\{f(x)\} = \frac{-(4b^2 - 8c^2)}{4} = (2c^2 - b^2)$$

And $\max\{g(x)\} = -\frac{(4c^2+4b^2)}{4(-1)} = (b^2 + c^2)$

Since, $\min f(x) > \max g(x) \Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow\ \ c^2 > 2b^2\ \ \Rightarrow\ \ |c| > \sqrt{2}|b|$

6 **(b)**

$$\frac{a+b+c}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} (\because \text{A. M.} \geq \text{H. M.})$$

$$\Rightarrow (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$$

7 **(d)**

$a + b + c = 18$

$$\Rightarrow 2 \times \frac{a}{2} + 3 \times \frac{b}{3} + 4 \times \frac{c}{4} = 18$$

Using weighted A.M. and G.M. inequality, we get

$$\frac{2 \times \frac{a}{2} + 3 \times \frac{b}{3} + 4 \times \frac{c}{4}}{9} \geq \left(\left(\frac{a}{2}\right)^2 \left(\frac{b}{3}\right)^3 \left(\frac{c}{4}\right)^4\right)^{1/9}$$

$$\Rightarrow 2^9 \geq \frac{a^2}{2^2} \times \frac{b^3}{3^3} \times \frac{c^4}{4^4}$$

$\Rightarrow ab^3c^4 \leq 3^3 \times 2^{19}$

8 **(b)**

Using A. M. ≥ G. M., we have

$x^4 + y^4 \geq 2x^2y^2$ and $2x^2y^2 + z^2 \geq \sqrt{8}xyz$

$$\Rightarrow \frac{x^4 + y^4 + z^2}{xyz} \geq \sqrt{8}$$

9 **(c)**

$$y = \frac{3^x}{3} + \frac{1}{3^x 3}$$

Now,

A.M.≥G.M.

$$\Rightarrow \frac{\frac{3^x}{3} + \frac{1}{3^x 3}}{2} \geq \sqrt{\frac{3^x}{3}\frac{1}{3^x 3}}$$

$$\Rightarrow 3^{x-1} + 3^{-x-1} \geq \frac{2}{3}$$

10 **(b)**

$$2\log_{10} x - \log_x 0.01 = 2\log_{10} x - \frac{\log_{10} 0.01}{\log_{10} x}$$

$$= 2\log_{10} x + \frac{2}{\log_{10} x}$$

$$= 2\left[\log_{10} x + \frac{1}{\log_{10} x}\right]$$

[Here $x\ > 1\ \Rightarrow\ \log_{10} x > 0$]

Now,

A.M. ≥ G.M.

$$\Rightarrow\ \frac{\log_{10} x + \frac{1}{\log_{10} x}}{2} \geq \left(\log_{10} x \frac{1}{\log_{10} x}\right)^{1/2}$$

$\Rightarrow \log_{10} x + \dfrac{1}{\log_{10} x} \geq 2$

11 **(c)**

We have,

$\dfrac{a^3+1}{2} \geq \sqrt{a^3 \times 1}$

and $\frac{a^2+a}{2} \geq \sqrt{a^2 a^1}$

Adding, we get

$\dfrac{a^3+a^2+a+1}{2} \geq 2\sqrt{a^3}$

12 **(c)**

Consider $1/l, 2/m, 3/n$ and use A.M. ≥ G.M. Then.

$\dfrac{1}{3}\left(\dfrac{1}{l}+\dfrac{2}{m}+\dfrac{3}{n}\right) \geq \left(\dfrac{1}{l}\dfrac{2}{m}\dfrac{3}{n}\right)^{1/3} = \left(\dfrac{6}{lmn}\right)^{1/3}$

But $lmn = 48$

$\therefore \dfrac{1}{3}\left(\dfrac{1}{l}+\dfrac{2}{m}+\dfrac{3}{n}\right) \geq \left(\dfrac{6}{48}\right)^{1/3} = \dfrac{1}{2}$

$\therefore \left(\dfrac{1}{l}+\dfrac{2}{m}+\dfrac{3}{n}\right)_{\min} = \dfrac{3}{2}$

13 **(d)**

$a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)$

$= ab^2+ac^2+bc^2+ba^2+ca^2+cb^2$

Using A.M. ≥ G. M. we get

$\dfrac{ab^2+ac^2+bc^2+ba^2+ca^2+cb^2}{6} \geq (a^6b^6c^6)^{1/6}$

$\Rightarrow a(b^2+c^2)+b(c^2+a^2)c(a^2+b^2) \geq 6abc$

14 **(b)**

A.M. ≥ G.M.

$\Rightarrow \dfrac{a+b+c}{3} \geq (abc)^{1/3}$

$\Rightarrow a+b+c \geq 3(abc)^{1/3}$

But given $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in A.P.

($\because abc \neq 0$). Hence,

$2abc = 1 + a^2b^2c^2$

$\Rightarrow (abc-1)^2 = 0$

$\therefore abc = 1$

Now from Eq. (i). we get

$a+b+c \geq 3(1)^{1/3}$

$\Rightarrow (a+b+c) \geq 3$

Hence, minimum value of $a+b+c$ is 3

15 **(d)**

We have A.M. ≥ G.M.

$\therefore \dfrac{a}{b}+\dfrac{b}{a}+\dfrac{b}{c}+\dfrac{c}{b}+\dfrac{c}{a}+\dfrac{a}{c} \geq 6$

$\Rightarrow \dfrac{b+c}{a}+\dfrac{c+a}{b}+\dfrac{a+b}{c} \geq 6$

Hence, the least value is 6

16 **(a)**

Using A.M. and H.M. inequality, we get

$\dfrac{2bc}{b+c} \leq \dfrac{b+c}{2} . \dfrac{2ac}{a+c} \leq \dfrac{a+c}{a} . \dfrac{2ab}{a+b+} \leq \dfrac{a+b}{2}$

$\Rightarrow \dfrac{bc}{b+c}+\dfrac{ac}{a+c}+\dfrac{ab}{a+b} \leq \dfrac{1}{2}(a+b+c)$

17 **(b)**

Since A.M. ≥ G.M. for different numbers, so

$\dfrac{(b+c-a)(c+a-b)}{2} > [(b+c-a)(c+a-b)]^{1/2}$

$\Rightarrow c > [(b+c-a)(c+a-b)]^{1/2}$

Similarly,

$b > [(b+c-a)(a+b-c)]^{1/2}$

and

$a > [(a+b-c)(c+a-b)]^{1/2}$

Multiplying, we get

$abc > (b+c-a)(c+a-b)(a+b-c)$

$\Rightarrow (b+c-a)(c+a-b)(a+b-c) - abc < 0$

18 **(a)**

A.M. ≥ G.M.

$\Rightarrow \dfrac{bcx+cay+abz}{3} \geq (a^2b^2c^2xyz)^{1/3}$

$\Rightarrow bcx+cay+abz \geq 3xyz$

or $bcx+acy+abz \geq 3abc$

19 **(a)**

a, b, c, d are in H.P. Hence, b is H.M. of a and c; c is H.M. of b and d. Using A.M. ≥ H.M., we get

$\dfrac{a+c}{2} > b \Rightarrow a+c > 2b$

and

$\dfrac{b+d}{2} > c \Rightarrow b+d > 2c$

$\Rightarrow a+c+b+d > 2b+2c$

$\Rightarrow a+d > b+c$

20 **(d)**

Let $a_1, a_2, \dots, a_n$ be n positive numbers such that $a_1 a_2 \cdots a_n = n^n$. Since A.M. ≥ G.M., hence,

$\dfrac{a_1+a_2+\ldots+a_n}{n} \geq (a_1 a_2 \ldots a_n)^{1/n}$

$\Rightarrow \dfrac{a_1+a_2+\ldots+a_n}{n} \geq n$

$\Rightarrow a_1+a_2+\ldots+a_n \geq n^2$

21 **(a)**

Let, $x-3=t$

$\Rightarrow x-2=(t+1)$ and $x-1=t+2$

$\Rightarrow f(x) = \dfrac{(x-2)(x-1)}{(x-3)}$

$= \dfrac{(t+1)(t+2)}{1}$

$= \dfrac{(t^2+3t+2)}{1}$

$= t + \frac{2}{t} + 3$

$\geq 3 + 2\sqrt{2}$ (using A.M. ≥ G.M., as $t > 0$)

22 **(d)**

$a^2 + b^2 = k^2$

$\left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2 = a^2 + b^2 + 4 + \frac{1}{a^2} + \frac{1}{b^2}$

$= k^2 + 4 + \frac{k^2}{a^2 b^2}$

$\geq k^2 + 4 + \frac{k^2}{\left(\frac{a^2+b^2}{2}\right)^2}$

$= k^2 + 4 + \frac{4}{k^2}$

23 **(c)**

a, b, c are in H.P. hence, H.M. of a and c is b

$\therefore \sqrt{ac} > b$ ($\because$ G.M. ≥ H.M.)

Since A.M. ≥ G.M., so

$\frac{a^{101} + c^{101}}{2} > \left(\sqrt{ac}\right)^{101} > b^{101} \left(\because \sqrt{ac} > b\right)$

$\Rightarrow 2b^{101} - a^{101} - c^{101} < 0$

Let,

$f(x) = x^2 - kx + 2b^{101} - a^{101} - c^{101}$

$\therefore\ f(0) = 2b^{101} - a^{101} - c^{101} < 0$

Hence equation $f(x) = 0$ has one root in $(-\infty, 0)$ and other in $(0, \infty)$

24 **(a)**

$2E = \frac{2a}{b+c-a} + \frac{2b}{c+a-b} + \frac{2c}{a+b-c}$

$= \frac{2a}{b+c-a} + 1 + \frac{2b}{c+a-b} + 1 + \frac{2c}{a+b-c} + 1 - 3$

$= (a+b+c)\left(\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}\right) - 3$

Using A.M. ≥ H.M. we have

$\frac{\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}}{3} \geq \frac{3}{a+b+c}$

$\Rightarrow (a+b+c)\left(\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}\right) \geq 9$

$\Rightarrow (a+b+c)\left(\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}\right) - 3 \geq 6$

$\Rightarrow E \geq 3$

25 **(b)**

Applying A.M.≥ G.M. in $6\tan^2\phi, 54\cot^2\phi, 18$, we get

$\frac{6\tan^2\phi + 54\cot^2\phi + 18}{3} \geq (6 \times 54 \times 18)^{1/3}$

≥ 18

Now equality holds when

$6\tan^2\phi = 54\cot^2\phi = 18$

$\Rightarrow \tan^2\phi = 3$ and $\cot^2\phi = \frac{1}{3}$

Hence, the statements I and II correct

26 **(a,b.)**

We have,

$2s = a + b + c$

$A^2 = s(s-a)(s-b)(s-c)$

Now, A.M. ≥ G.M.

$\Rightarrow \frac{s + (s-a) + (s-b)(s-c)}{4} \geq [s(s-a)(s-b)(s-c)]^{1/4}$

$\Rightarrow \frac{4s - 2s}{4} \geq [A^2]^{1/4}$

$\Rightarrow s/2 \geq A^{1/2} \Rightarrow A \leq s^2/4$

Also,

$\frac{(s-a) + (s-b) + (s-c)}{3} \geq [(s-a)(s-b)(s-c)]^{1/3}$

$\Rightarrow \frac{s}{3} \geq \left[\frac{A^2}{s}\right]^{1/3}$

$\Rightarrow \frac{A^2}{s} \leq \frac{s^3}{27}$

$\Rightarrow A \leq \frac{s^2}{3\sqrt{3}}$

27 **(a, b)**

Using A.M. ≥ G.M. one can show that

$(b+c)(c+a)(a+b) \geq 8abc$

$\Rightarrow (p-a)(p-b)(p-c) \geq 8abc$

Therefore, (b) holds, Also,

$\frac{(p-a) + (p-b) + (p-c)}{3} \geq [(p-a)(p-b)(p-c)]^{1/3}$

$\Rightarrow \frac{3p - (a+b+c)}{3} \geq [(p-a)(p-b)(p-c)]^{1/3}$

$\Rightarrow \frac{2p}{3} \geq [(p-a)(p-b)(p-c)]^{1/3}$

$\Rightarrow (p-a)(p-b)(p-c) \leq \frac{8p^3}{27}$

Therefore, (a) holds. Again,

$\frac{1}{2}\left(\frac{bc}{a} + \frac{ca}{b}\right) \geq \sqrt{\left(\frac{bc}{a}\frac{ca}{b}\right)}$

and so on. Adding the inequalities, we get

$\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq a + b + c = p$

Therefore, (c) does not hold

28 **(a,b,c)**

(a) $\because$ AM $\geq$ GM

$$\Rightarrow \frac{a_1^3+a_2^3+a_3^3}{3} \geq 3\sqrt{a_1^3a_2^3a_3^3}$$

$$\Rightarrow a_1^3+a_2^3+a_3^3 \geq 3a_1a_2a_3$$

(b) $\frac{\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_1}}{3} \geq 3\sqrt{\frac{a_1}{a_2}\times\frac{a_2}{a_3}\times\frac{a_3}{a_1}}$

$$\Rightarrow \frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_1} \geq 3$$

(c) $\because \frac{(a_1+a_2+a_3)}{3} \geq 3\sqrt{a_1a_2a_3}$

and $\frac{\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)}{3} \geq 3\sqrt{\frac{1}{a_1}\cdot\frac{1}{a_2}\cdot\frac{1}{a_3}}$

$$\therefore (a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right) \geq 9$$

(d) option (d) is not correct

29 **(c, d.)**

Consider the A.P. Since a is equidistant from the first term α and the last term β of the A.P., therefore, α, a, β are in A.P. Hence, a is the A.M. of α and β. So,

$$a = \frac{\alpha+\beta}{2}$$

Similarly, b and c are the geometric and harmonic means, i.e.,

$b = \sqrt{\alpha\beta}$ and $c = \frac{2\alpha\beta}{\alpha+\beta}$

Since A.M., G.M. and H.M. are in G.P. and A.M. $\geq$ G.M. $\geq$ H.M., therefore a, b, c are in G.P. and $a \geq b \geq c$

30 **(a, c)**

A.M. of x and z is y. G.M of x and z is $\sqrt{xz}$

Now. A.M. $\geq$ G.M. $\Rightarrow y^2 \geq xz$

Also, A.M. $\geq$ H.M. $\Rightarrow y \geq \frac{2xy}{x+z}$

$$\frac{x+y}{2y-x} = \frac{x+y}{x+z-x} = \frac{x+y}{z} \text{ and } \frac{y+z}{2y-z} = \frac{y+z}{x}$$

$$\therefore \frac{\frac{x+y}{2y-x}+\frac{y+z}{2y-z}}{2} \geq \sqrt{\frac{x+y}{z}\frac{y+z}{x}}$$

$$= \sqrt{1+\frac{y(x+y+z)}{xz}} \left[\because x+z=2y\right]$$

$$= \sqrt{1+\frac{3y^2}{xz}}$$

$$\therefore \frac{x+y}{2y-x}+\frac{y+z}{2y-z} \geq 2\sqrt{1+3\frac{y^2}{xz}}$$

$$\geq 4 \;\left[\because y^2 \geq xz\right]$$

32 **(c)**

Let roots of equation $x^6-12x^5+bx^4+cx^3+dx^2+ex+64=0$ be

$x, i = 1, 2, \ldots, 6$ Now,

$x_1+x_2+x_3+x_4+x_5+x_6 = 12$

and

$x_1x_2x_3x_4x_5x_6 = 64$

Thus,

$$\frac{x_1+x_2+\ldots+x_6}{6} = 2 \text{ and } (x_1x_2x_3x_4x_5x_6)^{1/6} = 2$$

$\Rightarrow$ A.M. = G.M.

$\Rightarrow x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 2$

Hence, the given equation is equivalent to

$(x-2)^6 = 0$

or

$$x^6-12x^5+60x^4-160x^3+240x^2-192x-64 = 0$$

$$\therefore f(1) = 1-12+60-160+240-192+64 = 1$$

33 **(d)**

For non-negative values of a, roots must be negative Let the roots be $x_1, x_2, x_3, x_4 (< 0)$. Then,

$x_1+x_2+x_3+x_4 = -a$

$x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4 = b$

$x_1x_2x_3+x_2x_3x_4+x_3x_4x_1+x_4x_1x_2 = -c$

$x_1x_2x_3x_4 = 1$

Now,

$$\frac{(-x_1)+(-x_2)+(-x_3)+(-x_4)}{4} \geq [(-x_1)(-x_2)(-x_3)(-x_4)]^{1/4}$$

($\because$ A.M. $\geq$ G.M.)

$$\Rightarrow \frac{a}{4} \geq 1$$

$$\Rightarrow a \geq 4$$

Hence, the minimum value of a is 4. Similarly,

$$\frac{x_1x_2+x_1x_3+\ldots+x_3x_4}{6} \geq [x_1^3x_2^3x_3^3x_4^3]^{1/4}$$

$$\Rightarrow \frac{b}{6} \geq 1$$

$$\Rightarrow b \geq 6$$

Hence, the minimum value of b is 6. Finally,

$$\frac{-x_1x_2x_3-x_2x_3x_4-x_3x_4x_1-x_4x_1x_2}{4} \geq [x_1^3x_2^3x_3^3x_4^3]^{1/4}$$

$$\Rightarrow \frac{c}{4} \geq 1$$

$$\Rightarrow c \geq 4$$

Hence, the minimum value of c is 4

34 **(8)**

Consider values xy, yz, zx

Now $\text{A.M.} \geq \text{G.M.}$

$\Rightarrow \dfrac{xy + yz + zx}{3} \geq (x^2y^2z^2)^{1/3}$

$\Rightarrow 4^{3/2} \geq xyz$

$\Rightarrow xyz \leq 8$

35 **(3)**

$a + b = 3$

HM $\leq$ AM for 3 numbers $\frac{a}{2}, \frac{a}{2}, b$ we have

$$\frac{3}{\frac{2}{a} + \frac{2}{a} + \frac{1}{b}} \leq \frac{\frac{a}{2} + \frac{a}{2} + b}{3} = 1;$$

$$\therefore\ 1 \geq \frac{3}{\frac{2}{a} + \frac{2}{a} + \frac{1}{b}} \Rightarrow \frac{2}{a} + \frac{2}{a} + \frac{1}{b} \geq 3$$

$$\therefore\ \frac{4}{a} + \frac{1}{b} \geq 3$$

36 **(2)**

As $x, y \in R$ and $xy > 0$, so x and y will be of same sign

Therefore, all the quantities $\frac{2x}{y^3}, \frac{x^3y}{3}, \frac{4y^2}{9x^4}$ are positive

Now A.M. ≥ G.M.

$$\Rightarrow \frac{2x}{y^3} + \frac{x^3y}{3} + \frac{4y^2}{9x^4} \geq 3\left(\left(\frac{2x}{y^3}\right)\left(\frac{x^3y}{3}\right)\left(\frac{4y^2}{9x^4}\right)\right)^{1/3}$$

$$= 3 \times \frac{2}{3} = 2$$

37 **(2)**

$$f(x) = \frac{4x^2 + 8x + 13}{6(1+x)}$$

$$= \frac{4(x+1)^2 + 9}{6(1+x)}$$

$$= \frac{2}{3}(x+1) + \frac{3}{2(x+1)}$$

$$\geq 2\sqrt{\frac{2}{3} \cdot \frac{3}{2}} = 2 \quad (\text{A.M.} \geq \text{G.M.})$$

Therefore, minimum value of $f(x)$ is 2

38 **(3)**

$9a + 3b + c = 90$

$\Rightarrow 3a + b + \dfrac{c}{3} = 30$

Now consider numbers $3a, b$ and $\frac{c}{3}$

$$\left(3a \times b \times \frac{c}{3}\right)^{\frac{1}{3}} \leq \frac{3a + b + \frac{c}{3}}{3} \text{ (as GM} \leq \text{AM)}$$

$\Rightarrow (abc)^{1/3} \leq \dfrac{30}{3} = 10$

$\Rightarrow abc \leq 1000$

$\Rightarrow \log a + \log b + \log c \leq 3$

39 **(6)**

Using AM ≥ GM for $x^2, 2xy, 2xy, 4y^2, z^2, z^2$

$$\therefore\ \frac{x^2 + 2xy + 2xy + 4y^2 + z^2 + z^2}{6} \geq [16(xyz)^4]^{1/6}$$

$= [16(32)^4]^{1/4} = (224)^{1/6} = 16$

$\Rightarrow m/16 = 6$

40 **(4)**

We have $\dfrac{2\left(\frac{x}{2}\right) + y}{3} \geq \left(\left(\frac{x}{2}\right)^2 y\right)^{1/3}$

$\Rightarrow \left(\dfrac{3}{3}\right)^3 \geq \dfrac{x^2y}{4} \Rightarrow x^2y \leq 4$

Therefore, maximum value of x^2y is 4